



# Nathaniel Bowditch

## Math, Science and Social Studies

### A Curriculum Designed for Grades 5-9

Curriculum developed by  
The Bowditch Initiative of Historic Salem Inc.  
in partnership with the House of Seven Gables Historic Site  
Salem, Massachusetts  
2001



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Funding provided by the Essex National Heritage Area, The Boston Marine Society,  
The Board of Commissioners of Trust Funds and The Marine Society At Salem

The Marine Society  
At Salem



The Boston  
Marine  
Society

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***[www. nathanielbowditch.org](http://www.nathanielbowditch.org)***



*...as a dramatic writer throws himself successively into the character of a drama, he is composing that he may express the ideas and emotions peculiar to each other, so the mind of a teacher should migrate, as it were, into those of his pupils to discover what they know, and feel and need; and then supply from his own stock what they require, he should reduce it to such a form and bring it within such a distance that they can reach it, seize it, and appreciate it.*

**Horace Mann, 1840**  
***The Art of Teaching, The Aptness to Teach***  
**p. 16,17**

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# NATHANIEL BOWDITCH



## INTRODUCTION

The story of Nathaniel Bowditch, mathematician, astronomer, navigator, and actuary, is the story of post-revolutionary America and the achievements of one man during this remarkable period in our nation's history. Nathaniel Bowditch, a self-educated, mathematically-gifted young man from Salem, Massachusetts, was born in 1773 into poverty. Although his early opportunities in life were extremely limited, Nathaniel's contributions to the new nation would later overshadow the difficulties of his childhood. For example, Bowditch's revision and recalculation of more than 8,000 mathematical errors in British navigation tables was a mammoth-sized task even now largely unheralded except by navigators. His publication of the revised tables in *The New American Practical Navigator* saved countless ships from fatal navigational errors, thus assuring their safe and swift arrival to their destinations. The now *American Practical Navigator* is in its 75th edition, and still relied upon by the United States Navy.

Bowditch displayed exemplary qualities in mathematics and science. His inquisitive mind sought patterns and solutions in everything around him. This search for order was particularly acute in his study of astronomy and navigation. He also displayed a flair for other languages, translating

Sir Isaac Newton's *Principia* from Latin to English, and later translating and revising Pierre La Place's *Mécanique Celeste* from French to English. These monumental works in physics and astronomy were now available to the new nation and its growing scientific community.

Nathaniel was also a natural teacher. His aptness for simplification of advanced mathematics achieved great acclaim from the sailors aboard his ships. Willing seamen were taught the complexities of celestial navigation, mastering even the difficult lunar calculations essential for determining longitude. The standard method for calculating lunars was greatly improved by Bowditch. These new lunar calculations of longitude aided countless sea captains, as the recently invented chronometer was still far too expensive for most captains to purchase.

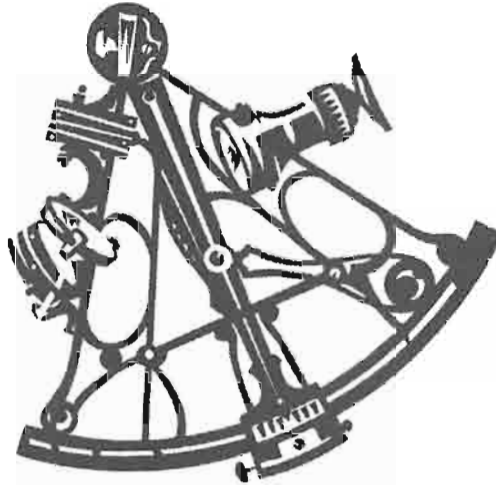
The life and accomplishments of Nathaniel Bowditch are reviewed and modeled in the following mathematics and science curriculum for middle school students. Four themes from his life-long accomplishments were selected to focus the math/science units. These unit themes are as follows: historical context, patterns, simplification, and self-education. The accompanying lesson plans include Bowditch and Salem, Bowditch the Mathematician, Patterns in Mathematics, Patterns in Astronomy, Patterns in Architecture, Math Simplification, Mathematical Skills and Habits, and Navigation.

Preparation of this curriculum revolved around the Massachusetts Curriculum Frameworks for grades 5-9. Mathematics, science, and social studies frameworks pertaining to each unit are provided. The units and lesson plans are interdisciplinary and unsequenced. Teachers may select one or all lessons in any order to best fit their curriculum needs. Some units can be expanded, particularly the astronomy unit. The analysis of star data and creation of a student almanac provide many more possibilities for lessons beyond the current material. These lessons lend themselves to technology. Students can create spreadsheets, graphs, and graphic presentations (example: Excel and PowerPoint) from the data provided.

Middle school students enjoy real-life stories of personal accomplishments. Advances in math and science are all based on the accumulation of personal stories of victories, upsets, disappointments, and unexpected discoveries. The Nathaniel Bowditch story is a narrative of our early nation and one talented individual who persevered to overcome poverty, subsequently contributing greatly to the lives of others. It is a story well worth adding to our nation's treasure chest of heroes.

## Unit 1:

# NATHANIEL BOWDITCH



## Overview for Teachers

### Unit Outline

#### Introduction

Nathaniel Bowditch (1773-1838) was one of Salem's outstanding scientific minds and respected citizens. His father, Habakkuk, was frequently destitute. The Bowditch children had few warm coats and were left to wear their summer shirts and jackets in the cold New England winter. On several occasions other children would taunt Nathaniel and his siblings. His response was to laugh back, assuming they were unable to stand the cold Salem weather.

At the age of twelve Nathaniel was indentured to Ropes and Hodges Ship Chandlery. This indentureship lasted for nine years. Although a formal education was now denied him, Nat taught himself mathematics, astronomy, navigation, Latin, and French. In addition, the outstanding scientific library of the Salem Philosophical Society was made available to him by an invitation from Dr. William Bentley, a minister and well-respected scholar in Salem. Nathaniel took great advantage of this opportunity to improve his education even more.

Nathaniel made four voyages as the captain's clerk and supercargo between 1795 and 1803. He studied the science of calculating longitude by determining the moon's position. After making countless observations and calculations, Bowditch discovered over 8,000 errors in Moore's standard British navigational tables. In 1802 he published a revised version called *The New American Practical Navigator*. His book was the crowning achievement of numerous successful endeavors. The book has been translated into a dozen languages and has remained the sailor's bible through 75 editions. No sailor in his right mind would go to sea without his "Bowditch".

Nathaniel's extraordinary navigation skills were known throughout the Salem maritime community. The successful voyage of *The Putnam* is a testimonial to his skill and mathematical confidence. On Christmas day in 1803, *The Putnam* was lying off the coast of Massachusetts. Only miles from Derby Wharf, the ship was trapped in a thick fog bank. The approach to Salem harbor was filled with numerous small islands and hidden ledges. On a clear day navigation was hazardous; dense fog was impassable. Other captains anchored and waited for the weather to clear. Nathaniel, now Captain and part-owner of *The Putnam*, trusted his mathematics and his knowledge of the harbor to bring his ship home to Salem. Nathaniel had only two readings at the sun with his sextant, pinpointing his approximate location in Massachusetts Bay. With this information, a compass, and accurate charts of the harbors' approaches, Bowditch felt confident he could get *The Putnam* safely home and in time for Christmas.

Bowditch used time-honored methods of log and line to calculate *The Putnam's* speed and lead lines to verify the water's depth. As *The Putnam* entered the harbor she passed Bowditch's Ledge. Legend had it that it was named for an ancestor of Bowditch who had sunk a vessel on it. He saw the faint light of the lighthouse on Baker's Island. *The Putnam* soon passed a familiar sight, Winter Island, and suddenly the crew of *The Putnam* was home safe and sound. Bowditch's unshakable faith in his own calculations proved that with accurate mathematics, one could go almost anywhere practically blindfolded.

The people of Salem could not believe their eyes when Bowditch, wet and gaunt came walking through the dark and foggy Christmas night. This seemingly miraculous feat of navigation made Bowditch a living legend in maritime circles. At his death, the Boston Marine Society paid the following tribute to Nathaniel Bowditch: *His intuitive mind sought and amassed knowledge, to impart it to the world in more easy forms.*

## **Salem Merchants**

The port of Salem built its fortunes on the eastern luxuries trade during the late 18th and early 19th centuries. The shipping of necessities was a secondary business in Salem. Dozens of goods were exchanged between Salem and the Orient. A typical Salem East Indiaman left the wharf with cod, tobacco, leather, candles, pork, shoes, lumber, hats, furniture, butter, beef, cloth, cheese, onions, European hardware and American rum. On the return voyage, Yankee captains like Bowditch obtained tea and fine silks from China, coffee from Arabia, pepper from Sumatra and cotton textiles from distant India. In addition, they often increased their profits by obtaining ivory, wine, and gold dust from Africa. Spices such as ginger, cinnamon and cloves were obtained from the Spice Islands (East Indies). Few captains engaged directly in the slave trade, however their indirect involvement came from providing the slave plantations with goods.

The profits of this world trade were often distributed through the community. For example, Elias Hasket Derby, a very wealthy shipowner, allowed his apprentices to put their savings into small "ventures" in foreign trade. He gave them each space on his vessels for the new ventures. Nathaniel Bowditch got his start in foreign trade with Derby's assistance.

### **Objectives:**

- Students will understand terminology important to the Salem maritime merchants.
- Students will retrace the voyages of Nathaniel Bowditch.
- Students will investigate the history/geography/economics of the voyages.
- Students will analyze the contribution of the Salem craftsmen to the success of the seaport.
- Student will learn to judge the process of building a wooden vessel during Nathaniel Bowditch's time.

### **Skills:**

- Students will develop economic reasoning skills.
- Students will learn to judge cause and effect in historical and economic contexts.
- Students will understand the skills needed to become proficient craftsmen, and in particular, boat builders.

**Vocabulary:**

- |                |                      |            |                |
|----------------|----------------------|------------|----------------|
| • barter       | • export             | • import   | • venture      |
| • supercargo   | • apprentice         | • foreign  | • seaman       |
| • luxuries     | • Yankee             | • cobbler  | • barber       |
| • blacksmith   | • pewter             | • tanner   | • silversmith  |
| • cabinetmaker | • miller             | • cooper   | • whitesmith   |
| • shipwright   | • joiner             | • caulker  | • half-model   |
| • keel         | • stem & stern posts | • trunnels | • teredo worms |

**Frameworks connections:****History and Social Science:****Strand 1: History**

Standard 1: Chronology and Cause, p. 79

Standard 2: Historical Understanding, p. 2

Standard 6: Interdisciplinary Learning, p. 93

**Strand 2: Geography**

Standard 7: Physical spaces of the earth, p. 94

Standard 8: Places and regions of the world, p. 96

Standard 9: The effects of geography, pp. 98-100

**Strand 3: Economics**

Standard 12: Economic reasoning, p. 108

Standard 13: American and Massachusetts  
Economic History, p. 111

# Unit 1 Lesson Plans



## Lesson 1: Nathaniel Bowditch and Salem

### Objectives:

- Students will understand terminology important to the Salem marine merchants.
- Students will retrace the voyages of Nathaniel Bowditch.
- Students will investigate the history/geography/economics of the voyages.

### Skills:

- Students will develop economic reasoning skills
- Students will learn to judge cause and effect in historical and economic contexts

### Vocabulary:

- |              |              |           |           |
|--------------|--------------|-----------|-----------|
| • barter     | • export     | • import  | • venture |
| • supercargo | • apprentice | • foreign | • seaman  |
| • luxuries   | • Yankee     |           |           |

### Materials:

- |                                     |                               |
|-------------------------------------|-------------------------------|
| • Nathaniel Bowditch – handout      | • Salem Merchants – handout   |
| • Terms to Know – handout           | • World Geography - handout   |
| • World Map with scale              | • Colored Pencils             |
| • Map of South East Asia            | • Geography/History – handout |
| • The Voyages of Nathaniel Bowditch |                               |

### Procedure:

1. Read the handouts, entitled Nathaniel Bowditch and Salem Merchants.
2. Answer the questions following the reading.
3. Research the definitions on the Terms to Know handout.

### **Group Activity**

4. Retrace the voyages of Nathaniel Bowditch by following the steps listed in the handout, World Geography.
5. Answer the questions on the voyages of Bowditch, Geography/History handout.

### **Independent Research**

6. A captain like Nathaniel Bowditch had many navigational tools to help him pinpoint his exact location. Choose one of the following navigational tools and prepare an oral report on the history and the significance of the tool.

- astrolabe
- celestial almanac
- chronometer
- compass
- octant
- sextant
- telescope





# NATHANIEL BOWDITCH



**N**athaniel Bowditch (1773-1838) was one of Salem's outstanding scientific minds and respected citizens. His father, Habakkuk, was frequently destitute. The Bowditch children had few warm coats and were left to wear their summer shirts and jackets in the cold New England winter. On several occasions other children would taunt Nathaniel and his siblings. His response was to laugh back, assuming they were unable to stand the cold Salem weather.

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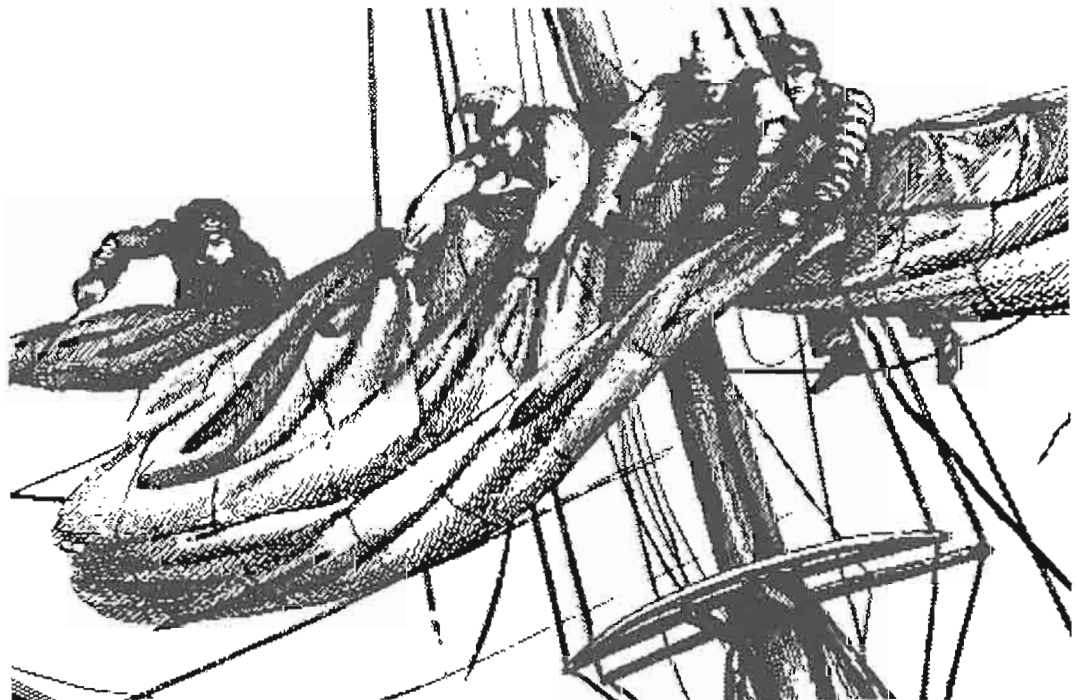
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# SALEM MERCHANTS

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# THE LIFE OF NATHANIEL BOWDITCH

1. How old was Nathaniel Bowditch when he died?
2. Describe Nathaniel's family when he was a child.
3. What did Nathaniel have to wear in the cold New England winters?
4. Between 1795-1803 how many sea voyages did Nathaniel Bowditch undertake?
5. How many errors did Bowditch discover in Moore's standard British navigational tables?
6. In what year did Bowditch publish his book, *The New American Practical Navigator*?
7. How many languages was his book translated into?
8. What nickname did sailors give his book?
9. What was the name of the ship in which Bowditch was captain?
10. On what day did Bowditch return home to Salem on a ship in 1803?
11. What tools did Bowditch use to guide his ship safely into Salem harbor? Name all five.
12. What were the weather conditions like when Bowditch navigated into Salem harbor?
13. Explain in your own words the following quote about Bowditch:  
*His intuitive mind sought and amassed knowledge, to impart it to the world in more easy forms.*



# WORLD GEOGRAPHY

## Retrace the Voyages of Nathaniel Bowditch

1. Choose five colored pencils. Assign a different color to each voyage.
2. Create a color key.
3. Locate and place Bowditch's destinations on your world map.  
(Don't forget Salem)
4. Now that you have chosen your colors proceed to retrace each of Bowditch's five voyages (see handout).
5. Locate and place all the oceans that Bowditch sailed across in his journey.
6. Label the Equator and the Tropics of Cancer and Capricorn.
7. Label the seven continents.
8. Label the mileage of each of Bowditch's voyages from Salem.
9. Label all ten seas found in South East Asia.
10. Label all current nations of South East Asia.

### Materials needed:

1. Color pencils
2. World map w/scale
3. Map of South East Asia
4. World atlas
5. The Voyages of Nathaniel Bowditch - handout



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# GEOGRAPHY/HISTORY

## Questions on the Voyages of Bowditch

1. Did Bowditch cross the equator during his third voyage?
2. Define Supercargo.
3. In Bowditch's time Jakarta, Indonesia was known as \_\_\_\_\_?
4. In what body of water is Réunion Island?
5. What was Réunion Island called during Bowditch's visit?
6. What country claims ownership of Réunion Island?
7. During the American Revolution, was this country on the American or British side?
8. What country did Bowditch visit during his third voyage?
9. What is the native language spoken in Cadiz?
10. What European colonial power controlled Batavia during Bowditch's visit on his fourth voyage?
11. Which continent is Indonesia a part of?
12. Indonesia today is ranked as the \_\_\_\_\_ most populated country?
13. What is the population of Indonesia today?
14. List the three spices found by merchants in Indonesia.
  1. \_\_\_\_\_
  2. \_\_\_\_\_
  3. \_\_\_\_\_
15. List several natural resources found in Indonesia today.



# THE VOYAGES OF NATHANIEL BOWDITCH

Ship	Dates	Destination
I. The Henry	1/11/1795- 1/11/1796	Isle of Bourbon (Réunion) Isle of France (Mauritius)
II. The Astrea	3/15/1796- 5/22/1797	Lisbon, Portugal Manila, Philippines
III. The Astrea	8/21/1798- 4/16/1799	Alicante and Cadiz, Spain
IV. The Astrea	7/23/1799- 9/15/1800	Batavia (Jakarta, Indonesia) and Manila
V. The Putnam	11/21/1802- 12/25/1803	Sumatra, Indonesia

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# TERMS TO KNOW

Look up the definition of the following terms.

1. Barter -
2. Export -
3. Import -
4. Venture -
5. Supercargo -
6. Apprentice -
7. Foreign -
8. Seaman-
9. Luxuries-
10. Yankee -





## Lesson 2: Tools of the Trade

### Objectives:

- Students will investigate and describe the uses of the sextant, the pendulum, the sundial, the quadrant, and the astrolabe.

### Skills:

- Students will develop independent research skills
- Students will utilize a variety of print and electronic media, including the Internet and the library.

### Vocabulary:

- Sextant
- Pendulum
- Sundial
- Quadrant
- Astrolabe

### Materials:

- Background information – Sextant, Pendulum, Quadrant, Astrolabe, and Sundial – handout
- Pattern for a three dimensional cube - handout

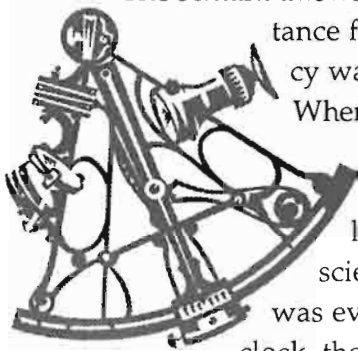
### Procedure:

1. Your task is to research 5 navigational instruments used during Nathaniel Bowditch's time: the sextant, the pendulum, the sundial, the quadrant, and the astrolabe. Some are actual navigational instruments; others were used as the framework for the instrument.
2. Research both the history and uses of each instrument. Include the earliest use, it's use during Bowditch's lifetime (1773-1838), and modern applications.
3. Include a diagram or picture of each instrument.
4. List all references, both print media and Internet.
5. Enlarge the pattern for a three dimensional cube to make a larger block.
6. Your research for each of the five instruments should include your name at the top of the first page, and a bibliography in the back.

## Background Information:

### Sextant, Pendulum, Quadrant, Astrolabe, and Sundial

**Sextant:** an optical instrument used for measuring the angular distance between any two objects. The English mathematician John Hadley and the American inventor Thomas Godfrey invented it about 1730. The navigator can measure the angular elevation of the sun and other celestial bodies and from this information calculate latitude and longitude. The optical system consists of a telescope and two mirrors, one fixed and one moveable. In the diagram, the telescope is mounted in a fixed position on the body of the instrument pointing toward the mirror; the top half of this mirror is transparent and the bottom half is silvered. A second mirror is angled above the first. An observer looks through the telescope and sees the horizon through the unsilvered portion of the mirror and at the same time sees the image of the star or the sun on the silvered portion as re-reflected from the second mirror. A sextant reading can be obtained that is double the actual altitude of the star.



The sextant allows you to determine latitude, or north/south distance from the equator, with accuracy. The same accuracy was needed for longitude, or the east/west distance. When a fleet of four ships ran aground in the fog and killed 2000 men, a large sum of money was offered as a prize for finding a way to calculate longitude. Isaac Newton was one of the famous scientists who tried to find a method. The problem was eventually solved by John Harrison with an accurate clock, the chronometer. By measuring the difference in local time versus the Greenwich time, you can calculate the distance east/west.

### Pendulum

In 1657 the Dutch physicist Christian Huygens demonstrated how a pendulum could be used to regulate a clock. In 1667 the English physicist Robert Hooke invented an escapement that allowed a smaller arc of oscillation. This escapement was improved upon by British clockmaker George Graham. John Harrison, the inventor of the chronometer, developed a means to compensate for a variation in the length of the pendulum due to temperature change.

### Quadrant

The sum of the height of the observer plus the product of the  $\tan(\text{angle})$  and the distance the object is from the observer = height of the object.

If the angle is 45, then the height of the object is equal to the distance the observer is from the object

## **Sundial**

Until the end of the 19th century, all time was local and related to the sun. In 1884 an international convention was held in Washington, DC to agree on a world wide system of time. A sundial consists of a dial plate which is marked with hour lines and a "gnomon", a projection that casts a shadow. The inclined edge of the gnomon is called the style; it is oriented parallel to the Earth's axis and points to a celestial location close to Polaris. Directions for making a sundial can be found at <http://cpcug.org/user/jaubert/jsundial.html>.

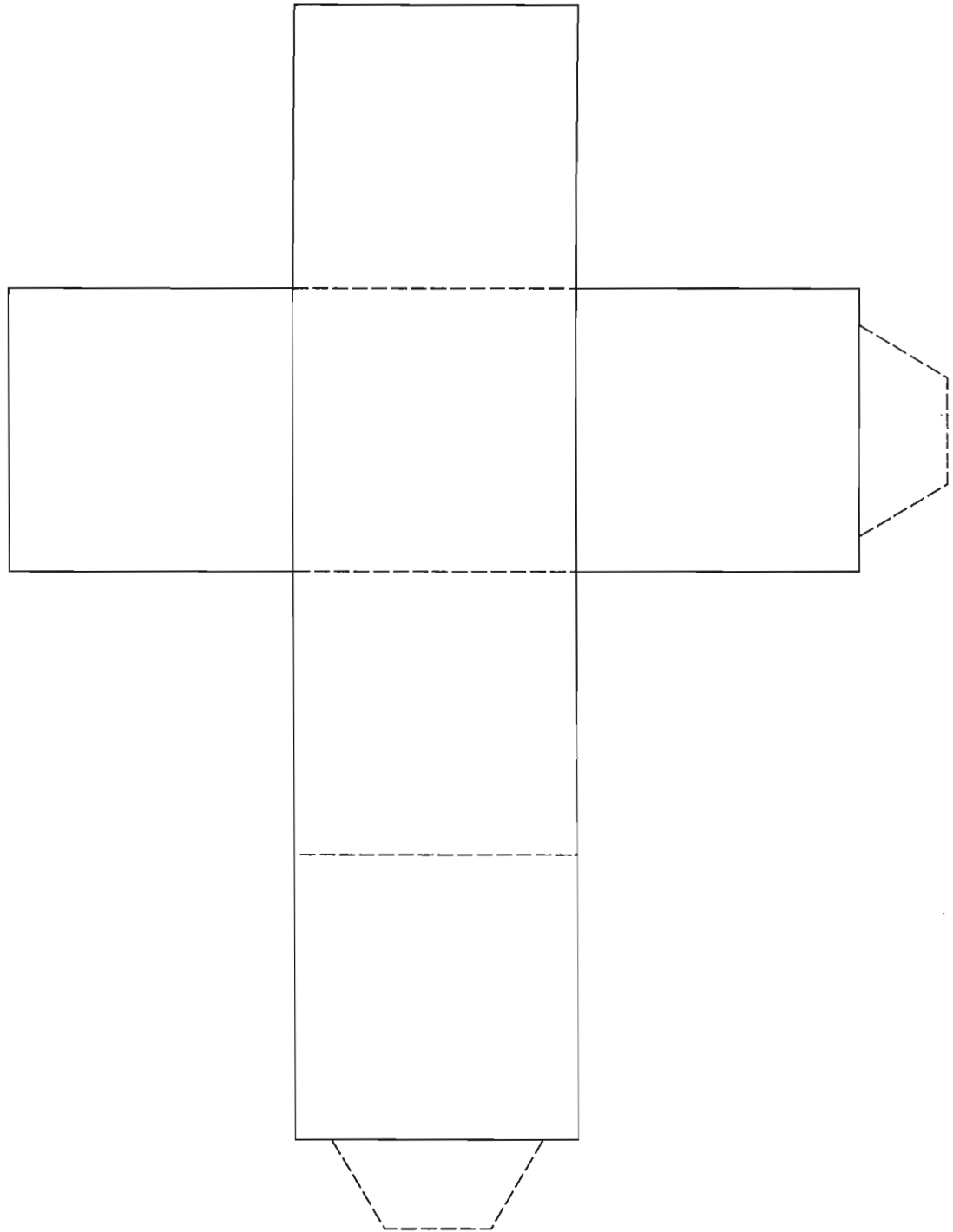
## **Astrolabe**

Astrolabes have been traced to the 6th century and came into wider use in Europe and the Islamic World during the Middle Ages. By mid 15th century, the astrolabe was adopted by mariners and used in celestial navigation. The mariner's astrolabe was replaced by the sextant. Typically, a sailor would identify the constellations visible in the sky around the sun at sunrise. At noon, the sailor would hold the astrolabe waist high and record how many degrees the sun is above the horizon. Using the data from both observations and the Rules for the Astrolabe, the sailor could look up the latitude of the ship.





# PATTERN FOR 3D BOX



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## Lesson 3: Craftsmen of Salem:

### Objectives:

- Students will understand the support system for the Salem economy provided by the local craftsmen.

### Skills:

- Students will relate nautical terms to occupations.

### Vocabulary:

- |                |              |               |
|----------------|--------------|---------------|
| • cobbler      | • barber     | • blacksmith  |
| • pewter       | • tanner     | • silversmith |
| • cabinetmaker | • miller     | • cooper      |
| • whitesmith   | • shipwright |               |

### Materials:

- Craftsmen of Salem – handout #1
- Craftsmen of Salem – handout #2
- Glossary of Sea Terms

### Procedure:

1. Read Craftsmen of Salem #1
2. Complete the handout, Craftsmen of Salem #2

### Group Activity:

3. Create flashcards with words and definitions of the glossary of sea terms words.
4. Challenge each group to design a board game utilizing the glossary words and the craftsmen labels.





## CRAFTSMEN OF SALEM #1

The Craftsmen of Salem were critical to the success of its merchants. This group of highly skilled tradesmen supplied the merchants with finished products that could be exchanged for the luxury goods obtained from ports all over the world. They were also instrumental in building and outfitting the Salem ships. For example, an East Indiaman slipping through the harbor was the result of a highly cooperative enterprise. The skills of at least 20 craftsmen were displayed in the finished vessel.

Salem possessed numerous shipyards during Bowditch's lifetime. Shipbuilders turned out mostly small fishing and coasting vessels. Salem did not build the largest vessels, but its East Indiamen, heavily armed merchant ships built for the East India trade, were equal to any in quality. East Indiamen, like those sailed by Bowditch during his voyages, were built at the boat yards located near the mouth of the South River. The Briggs'shipyard is located on Stage Point, where the present day Shetland facility stands. Another important shipyard was Becketts located on the outer harbor. Becketts was founded in the mid-17th century and was one of the earliest boat yards in Salem. It was owned by the Becket family and located at the foot of Becket Street, near the present-day power plant.

Some shipbuilders owned other businesses. The Briggs family owned a rope-walk to ensure an adequate supply of cordage for their shipbuilding endeavors. The rope-walk was located over pilings jutting out from present-day Briggs Street over Collins Cove.

During his indenture at the Ropes and Hodges Ship Chandlery (1785-94) Nathaniel learned all there was to know about the supplying of ships. In sea-ports, a chandlery provided everything a ship could possibly need from rope to marlin spikes and barrels of hard-tack. In this capacity the young Bowditch frequently conversed with the craftsmen and vendors of various nautical goods. This experience was to become crucial to his later success as a supercargo, and later as a captain or merchant.



# CRAFTSMEN OF SALEM #1

Below is a list and description of tradesmen found in Salem during Bowditch's lifetime. Match the following trades to their description.

- |                 |                                                       |
|-----------------|-------------------------------------------------------|
| A. Cobbler      | 1. A person who makes leather goods.                  |
| B. Barber       | 2. A person who builds ships.                         |
| C. Blacksmith   | 3. A person who makes barrels for storage.            |
| D. Pewter       | 4. A person who makes or repairs shoes.               |
| E. Tanner       | 5. A person who makes furniture.                      |
| F. Silversmith  | 6. A person who makes wigs.                           |
| G. Cabinetmaker | 7. A person who makes and repairs items of tin.       |
| H. Miller       | 8. A person who makes nails and other items of iron . |
| I. Cooper       | 9. A person who grinds wheat into flour.              |
| J. Whitesmith   | 10. A person who makes expensive dinnerware.          |
| K. Shipwright   |                                                       |



## Lesson 4. Building A Wooden Ship.

### Objectives:

- Student will examine the process of building a wooden vessel during Nathaniel Bowditch's time.

### Skills:

- Students will understand the skills need to become proficient craftsmen, and in particular, boat builders.

### Vocabulary:

- |              |                |                        |
|--------------|----------------|------------------------|
| • Half-model | • Mold-loft    | • Brad axe             |
| • Adze       | • Joiners      | • Keel                 |
| • Frames     | • Hull         | • Stem and stern posts |
| • Caulkers   | • Teredo worms | • Copper sheathing     |

### Materials:

- Building a Wooden Ship – handout
- Building a Wooden Ship – questions.

### Procedure:

1. Read building a Wooden Ship
2. Answer the questions in the handout.
3. Arrange field trips to the many Essex County ships and boat building sites.





# BUILDING A WOODEN SHIP

The creation of a wooden sailing ship such as Bowditch's *Putnam* began with a "half-model" or "lift-model". The process was accomplished by carpenters in an area called the "mold loft". The half model was separated into sections on the mold loft floor. The shape of the model was transferred to patterns or templates. Workmen shaped the frames of the ship with broad axes and adzes, using the patterns as guides. Natural curves of trees formed specific parts of the vessel's frame. Two favored woods for the frame and hull were white oak and live oak. These were favored for boat building due to their resistance to rot. Masts and spars, usually made of white pine, were pickled in saltwater ponds to both preserve them and to increase their resilience.

Carpenters, known as joiners, laid the keel, the great spine of the ship running along the bottom of the hull. The keel was fashioned from two or more pieces of wood "scarfed" together. The stem and stern post were then attached to the finished keel, which was lying on large blocks. The giant ribs of the ship, the frames, were raised next. The frames, together with the horizontal deck beams and the vertical stanchions, formed the contours of the hull. Together they provided a strong skeleton. All of these parts were held together by large wooden nails, or "trunnels". Trunnels were eventually replaced by bolts and spikes made from iron and copper.

The hull was caulked or sealed by caulkers, the highest paid of the ship building "mechanics". The planking seams were sealed with tarred hemp fibers, known as oakum. Joiners would smooth and plane the wood surface of the hull. Eventually the hull was sealed with a mixture of tree resin, sulfur, and tallow to repel boring teredo worms and barnacles. Once the hull was launched, it would swell in the saltwater thus helping to keep the new vessel somewhat watertight. During Bowditch's time, very wealthy shipowners covered the bottoms of their vessels in copper sheathing. Copper sheathing was very effective in protecting the hull from teredo worms and other destructive elements of the sea. Warships like the U.S.S. Constitution had copper sheathing to increase operational life. A well-built ship could be expected to last at least 20 to 30 years, if maintained properly.



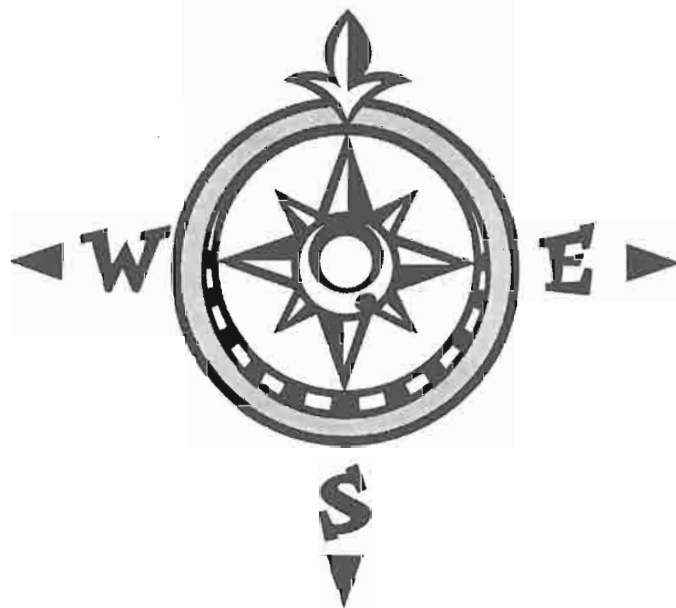
# BUILDING A WOODEN SHIP

1. What is the first step in building a wooden vessel?
2. What type of craftsmen began the first step of construction?
3. What kind of woods were the favored type to construct a ship?
4. Why were these woods favored?
5. What kind of wood were the masts and spars usually constructed of?
6. Why was the ship built at an angle?
7. The spine or backbone of the ship is called a \_\_\_\_\_.
8. The \_\_\_\_\_ acted like giant ribs providing a strong skeleton for the ship.
9. How did the carpenters fasten all the pieces of the ship together? Did they use iron and copper bolts and spikes?
10. Caulkers were the \_\_\_\_\_ paid of the mechanics that worked on ships.
11. What material did caulkers use to seal the seams between the wooded planks?
12. A mixture of \_\_\_\_\_ was used to protect the ship from teredo worms and barnacles.
13. If a merchant could afford to, he could use \_\_\_\_\_ metal to cover the bottom of his ship to protect it from living organisms.
14. How many years could an average ship last?



## UNIT 2:

# NATHANIEL BOWDITCH AS MATHEMATICIAN



## Overview for Teachers

### Unit Outline

#### Introduction:

Nathaniel Bowditch was a mathematician, an astronomer, a navigator, and an actuary. Mathematics and science were the center of his life in post-American Revolutionary Salem. At the time of his death in 1838, Nathaniel had earned membership into most of the scientific societies of the world:

- American Academy of Arts and Sciences, Boston (President)
- American Philosophical Society, Philadelphia
- Connecticut Academy of Arts and Sciences, New Haven
- Royal Society of London (England)
- Royal Irish Academy, Dublin (Ireland)
- Salem Marine Society, Salem, MA
- Royal Astronomical Society, London (England)
- Royal Academy of Palermo (Italy)
- Royal Academy of Berlin (Germany)

Nathaniel's genius was not necessarily in his new discoveries in math and science, but his gift for explaining complex subjects in terms understandable to the average citizen. His scientific writings were reviews and revisions of other, more complex texts. Bowditch would characteristically add additional tables, drawings, and explanations to improve the clarity and presentation of the original document.

Additionally, Nathaniel Bowditch the mathematician is seen in navigation circle as an unheralded genius, an important contributor to American freedom through sea power (Hubbard, 2000). Bowditch's *New American Practical Navigator* saved uncountable ships from certain destruction and insured arrival swiftly to port. This was all due to an advanced mathematical ability and scientific acuity.

The following lesson allows students to "put on a mathematician's shoes" to experience life as both a mathematician and an individual in the 18th or 19th century. Many prominent mathematical scholars are introduced for student research and characterization.

**Objectives:**

- Students will experience life as both a mathematician and an individual in the 18th or 19th century by "putting on mathematician's shoes."

**Skills:**

- Students will understand and apply scales.
- Students will use scale to develop a time line
- Students will sequence mathematicians' life times
- Students will research the contributions of famous mathematicians

**Vocabulary:**

- proportion

## **Frameworks Connections: Mathematics**

### **Mathematics:**

#### **Strand 2: Patterns, relations and functions**

- Describe, extend, create a wide variety of patterns, 2.4, p. 60

### **Science and Technology:**

#### **Strand 1: Inquiry**

- Note and describe relevant details, patterns, relationships, p.28

#### **Strand 2: Domains of Science, Life Sciences**

- Present evidence on how one species depends on another, p. 63

### **History and Social Science:**

#### **Strand 1: History**

##### **Standard 1: Chronology and cause**

- Understand chronological order of historical events, p. 78
- Understand meaning, implications, importance of historical events, p. 79.

##### **Standard 3: Research, evidence, and point of view**

- Acquire ability to frame questions that can be answered by historical study, p. 84

##### **Standard 6: Interdisciplinary Learning: Natural Science, Mathematics and Technology in History**

- Will describe and explain major advances over time, p. 92

## Unit 2 Lesson Plans



### Lesson 1: Nathaniel Bowditch as Mathematician

#### Objective:

- Students will experience life as both a mathematician and an individual in the 18th or 19th century by "putting on mathematician's shoes".

#### Skills:

- Students will need to develop a scale for their timeline.

#### Vocabulary:

- Proportion

#### Materials:

- Black line master tree cross section as an overhead transparency
- Time Line Directions for Activity 1 - handout
- Discussion Questions for Activity 2 - handout
- List of 18th Century and 19th Century mathematicians
- Name cards cut from black line master
- Long strips of paper (adding machine tape from old adding machines)
- Pencils, markers, rulers

#### Procedure:

##### Activity 1: Tree Ring Birthday and Time-Line Building

- Use the black line master of the tree trunk cross section to make one or two overhead transparencies. The projection must be large enough to estimate the location of the tree rings that correspond to the mathematician's birth day. If you make two transparencies, use one cross-section for each century. [Option 2: Use the black line master to have an enlarged poster size tree trunk made at a graphics studio.]
- Photocopy the names of the mathematicians onto a 60 pound weight card stock paper or similar weight paper. Cut out the strips of names on cut lines placing the strips into a basket or hat for students to draw their mathematician's name. Distribute the names to students. If you wish to use only selected names, we suggest that you choose mathematicians in the same century.

- Each group should have one roll of adding machine tape. Groups will need pencils, at least one marker, one or two rulers, and a sheet of directions for making a time line (included in this guide).
- Student groups go to the tree trunk on the transparency wall or poster and place their mathematician's name at a tree ring corresponding to the mathematician's birthday. Assume one tree was planted in 1700 and the other tree planted in 1800.
- While one group is at the tree cross section, the rest of the groups are deciding where their mathematician would be placed and starting the adding machine time line (see Time Line Directions for Activity 1).

## **Activity 2: Research**

### **Materials:**

- Name cards cut from black line master
- Optional: students dress in period attire
- Discussion Questions for Activity 2.

### **Prepare before the lesson:**

Assemble biographies of the mathematicians from either the listed websites, the AIMS resource books, or from other library sources. Students can prepare these biographies themselves after they choose their mathematician's name. This can happen during class time or as a homework assignment.

### **Procedure:**

**Cooperative Groups:** After students have selected their names, they go to the cooperative group listing the appropriate century for the person's birth date. No more than five students are allowed in any one group. The students will first read about the mathematician that they chose and then respond to discussion questions on the Discussion Questions for Activity 2 provided with this lesson.

Allow 15 minutes for the groups to complete the Discussion Questions for Activity 2. The selected speaker from each group will then share the group's research findings with the rest of the class.





# TIME LINE DIRECTIONS FOR ACTIVITY 1:

1. Your time line will cover a 100 year period. Create a scale for your time line so that all of the mathematicians in your group will fit the time line even though the dates may be between the inch marks.

Do your calculations here:

2. After you have decided how long your time line will be, unroll the correct amount from the adding machine roll.
3. Draw a line lengthwise down the center of the tape.
4. Draw the inch marks. Label the years that each inch mark represents (use the scale your group calculated in step 1).
3. Place the mathematician's name cards on the time line at their year of birth.
5. If your group finishes early, discuss how the mathematicians may have talked to each other.
6. If \_\_\_\_\_ had communicated with \_\_\_\_\_, how might that have changed their thinking?
7. Example: If Nathaniel Bowditch could have spoken to Isaac Newton, how would Bowditch's thinking have changed?
8. What topics would they have discussed?
9. How would they have communicated? by phone? by mail?

Write your ideas in the space below:



## DISCUSSION QUESTIONS FOR ACTIVITY 2:

1. List the mathematicians in your group in the chart below. Write a brief description about the work of each. Use one or more of these words as a category for the mathematician's work: geometry, algebra, number patterns, physics applications, statistics.

2. Did any of these mathematicians live during the same time? Write the names and the ages of each of your mathematicians at the time when Nathaniel Bowditch was born in 1773.

3. If your group finishes early, talk about how the mathematicians may have talked to each other.

What topics would they have discussed?

How would they have communicated?

Write your ideas in the space below. Use the back of this sheet if you need more room.

4. If any mathematician had already died, what did the current mathematicians probably learn from his/her earlier work?



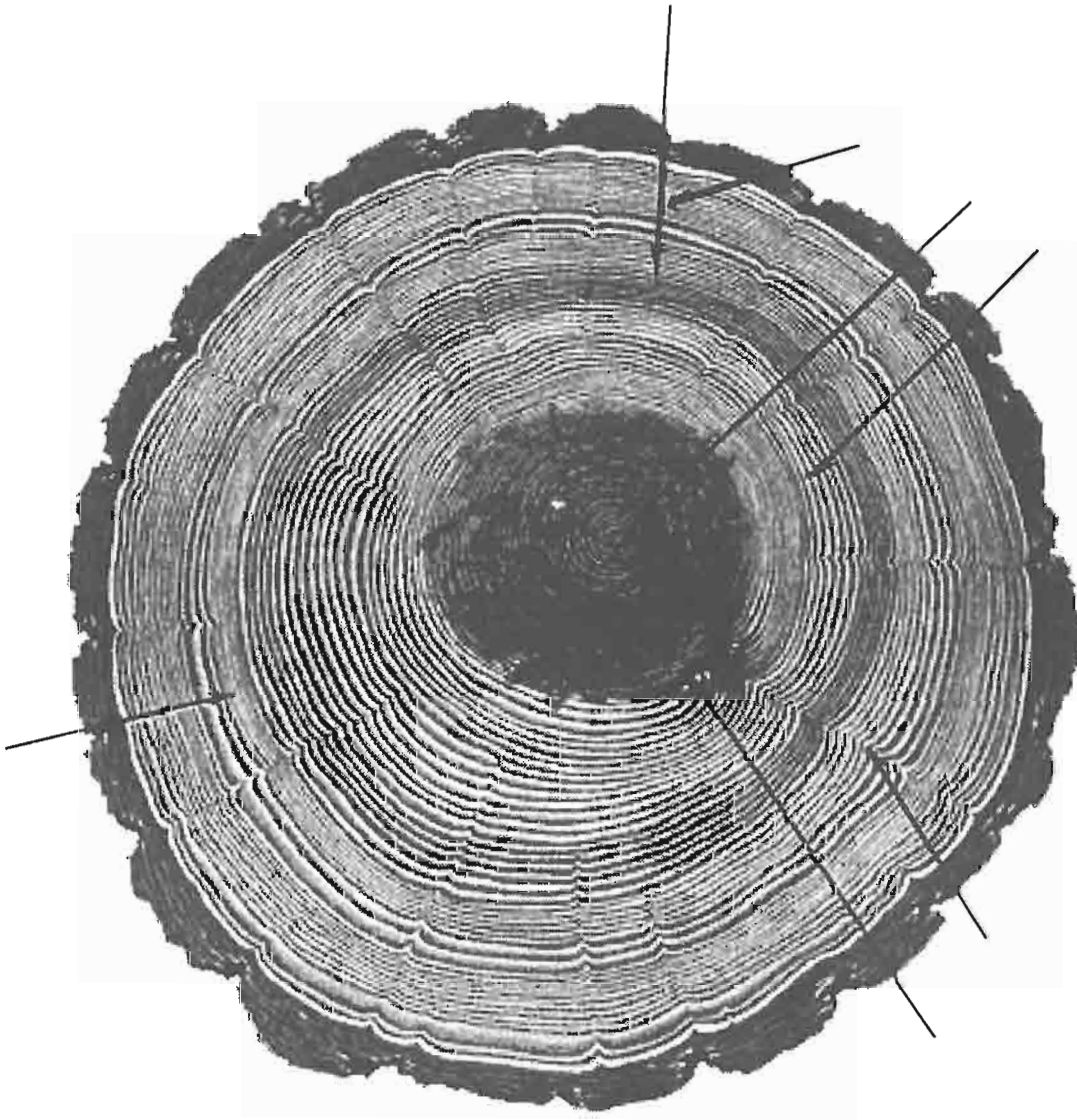


# RUBRIC FOR TEACHERS TO RATE GROUP ACTIVITY

## Time Line

Mathematician's Name	Nationality, Date of Birth Date of Death	Category of Major Work	Brief Description of Work

Name	Age when Bowditch was born



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# MATHEMATICIANS FROM THE 18TH & 19TH CENTURY

18th Century	19th Century
18th Century	19th Century

Choose mathematicians from these 18th century or 19th century mathematicians. Add others if you wish.

Joseph-Louis Lagrange 1736 - 1813	George Simon Ohm 1789 - 1854	Ludwig Otto Hesse 1811 - 1874
Mary Fairfax Somerville 1780 - 1872	August Ferdinand Mobius 1790 - 1868	Robert Woodhouse 1773 - 1827
Caroline Herschel 1750 - 1848	Charles Babbage 1792 - 1871	Carl Gustav Jacob Jacobi 1804 - 1851
Marie-Sophie Germain 1776 - 1831	Olinde Rodrigues 1794 - 1851	Evariste Galois 1811 - 1832
Benjamin Banneker 1731 - 1806	Johann Christian Doppler 1803 - 1853	George Boole 1815 - 1864
Leonhard Euler 1707 - 1783	Pierre Simon Laplace 1749 - 1827	Jules Antoine Lissajous 1822 - 1880
Nathaniel Bowditch 1773 - 1838	Ruan Yuan 1764 - 1849	Fofia Kovalevskaya 1850 - 1891
Sir Isaac Newton 1643 - 1727	Maria Gaetanna Agnesi 1718 - 1799	Ada Byron Lovelace 1815 - 1852
Carl Frederich Gauss 1777 - 1855	Jean Bernoulli 1744 - 1807	Florence Nightingale 1820 - 1910
Furukawa Ken 1783 - 1838	Wada Yenzo Nei 1787 - 1840	Mary Everest Boole 1832 - 1916
Nicolai Ivanovich Lobachevsky 1792 - 1856	George Peacock 1791 - 1858	John Wilson 1741 - 1793

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## EXTENSION ACTIVITY IDEAS

1. Students can pair up and interview each other as their mathematician talking across centuries. Pair students by common math ideas after reading biographies.
2. Students can write an interview with their own mathematician and create newspaper articles for a class publication. Students can name the newspaper and design the masthead.
3. Students can find major historical events occurring at the same time period and place these on cards for the time line.
4. Students can identify literary writers, artists and presidents living during their time line period, and create cards for each to place on the time line.
5. Students can research other important Salem citizens and place their name on cards for the time line.



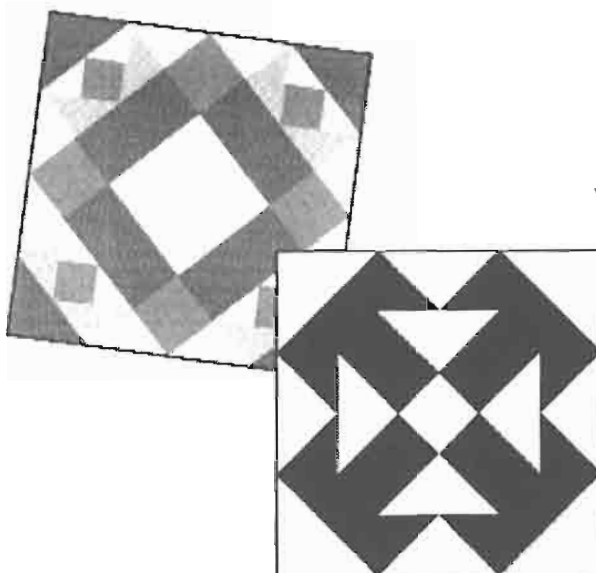
# RUBRIC FOR TEACHERS TO RATE GROUP ACTIVITY

Assessment objectives	1 Excellent	2 Good	3 Fair	4 Poor
1. Students agreed on a time scale				
2. Students completed the time line				
3. Students correctly placed names on time line				
4. Students completed the group sheet				
5. Students reported to class				
6. All students participated in activity				



## UNIT 3:

# PATTERNS



## OVERVIEW FOR TEACHERS

### Unit Outline

#### Introduction:

*'Every year since you were eight, you've gotten twice as pretty as you were the year before.'*

*'Hmmm...' Lizza bit her tongue and rolled her eyes. "I'm nineteen now...eight from nineteen..." She sighed. 'You mathematician! I wish you could at least pay a compliment without arithmetic! Eight from nineteen is eleven. Twice as pretty every year...Goodness, I'm twenty-two times as pretty!'*

*Nat laughed. 'You're two thousand and forty-eight times as pretty! Keep count on your fingers as I double one eleven times: two---four---eight---sixteen---thirty-two---sixty-four---one hundred twenty-eight---two hundred fifty-six---five hundred twelve---one thousand and twenty-four---and two thousand and forty-eight!'*

*'And next year...' Lizza said.*

*'Four thousand and ninety-six times as pretty!'*

*(Nathaniel Bowditch to his sister Lizza in **Carry On, Mr. Bowditch**, p. 70)*

Patterns in mathematics and astronomy fascinated Nathaniel Bowditch. The excerpt above illustrates his fondness for mathematical riddles, number sequences, and quick computations. Our Unit 3 Theme: Patterns, was selected to highlight Nathaniel's fascination with natural rhythms in both mathematics and astronomy. Patterns in architecture compliment the overall theme and extends the theme to the Bowditch home in Salem.

Our theme begins with Patterns in Mathematics. The first five lesson plans introduce students to problem solving, quilt designs, and the Fibonacci series. Students are provided opportunities to discover, explore, analyze, and predict mathematical patterns in the world around them.

**Objectives:**

- Students will demonstrate their recognition of numerical patterns by stating the rules that generate the patterns.
- Students will examine special patterns---namely, square numbers, triangular numbers, and cubes---and study their interrelationship.
- Students will find patterns within patterns.
- Students will study the famous Fibonacci sequence and discover examples in mathematics, nature and aesthetics.

**Skills:**

- Students will learn to use simple exponential notation to express numbers in a pattern generated by powers.

**Vocabulary:**

- patterns
- square number
- triangular number
- cube number
- exponent
- power
- consecutive



## **Frameworks connections:**

### **Mathematics**

#### **Strand 1: Number Sense**

**Standard 1.6:** Number and Number Relationships (p. 40)

Represent and use equivalent forms of numbers,  
including exponents.

Represent numerical relationships in graphs.

**Standard 1.7:** Number Systems and Number Theory  
(p.41)

Use operations involving integers and rational numbers.  
Demonstrate how basic operations are related to one  
another.

Create and apply number theory concepts.

**Standard 1.8:** Computation and Estimation (p.42)

Compute with whole numbers, fractions, decimals.

Use computation, estimation, and proportion to  
solve problems.

#### **Strand 2: Pattern Relations and Functions**

**Standard 2.4:** Patterns and Functions (p. 60)

Describe, extend, analyze, and create a wide variety  
of problems.

Describe and represent relationships with models, tables,  
graphs, rules.

**Standard 2.5:** Algebra (p.61)

Represent number patterns with tables, graphs, verbal  
rules and explore the interrelationship of these  
representations.

Analyze tables and graphs to identify properties  
and relationships.

#### **Strand 3: Statistics and Probability**

**Standard 4.3:** Statistics (p.90)

Collect, organize, and describe data systematically.

Construct, read, and interpret tables, charts, and graphs.

## Unit 3 Lesson Plans



### Lesson 1: Patterns Begin Simply

#### Objectives:

- Students will demonstrate their recognition of numerical patterns by stating rules that generate the patterns.
- Students will examine special patterns---namely square numbers and triangular numbers and their interrelationship.

#### Skills:

- Students will be able to find patterns within patterns.
- Students will know how to use patterns as a problem-solving technique.
- Students will learn to create their own numerical patterns and ask classmates to name the rules used to generate the patterns.

#### Vocabulary:

- pattern
- square number
- triangular number

#### Materials:

- pencils, paper, calculator

#### Procedure:

1. Distribute worksheet "Patterns Begin Simply".
2. Each students must predict the next three numbers of each pattern and discuss the rule which allows them to predict the patterns.
3. Pair off the students. Each student must make up a pattern (the possibilities are endless!) and have his partner guess the rule and how the pattern will continue.
4. Distribute worksheet "Using a Pattern to Solve a Problem More Quickly".

#### Handouts:

- Worksheet: "Patterns Begin Simply" (4pp.)
- Worksheet: "Using a Pattern to Solve a Problem More Quickly" (1p.)

## Patterns Begin Simply

As you have probably noticed, patterns are everywhere---in a snowflake, a pine cone, a sunflower, on wallpaper, the kitchen floor, a quilt. Patterns are both beautiful and useful. What makes a pattern?

**Definition:** Pattern - a design or set of features that repeats in an orderly way or that has a rule to help you predict what will happen next.

Mathematics has patterns, too, and that makes mathematics predictable. You can figure out what will happen next and that can help you solve some difficult problems!

Let's look at a few simple mathematical patterns. Figure out a rule which might have been used to create each pattern and tell which numbers should go on the blanks.

1, 2, 3, 4, 5, \_\_\_\_ \_\_\_\_ \_\_\_\_ (6, 7, 8)  
(Rule: Add 1 each time.)

2, 4, 6, 8, \_\_\_\_ \_\_\_\_ \_\_\_\_ (10, 12, 14)  
(Rule: Add 2 each time.)

100, 95, 90, 85, 80, \_\_\_\_ \_\_\_\_ \_\_\_\_ (75, 70, 65)  
(Rule: Subtract 5 each time.)

1, 2, 4, 8, 16, 32, 64, 128, \_\_\_\_ \_\_\_\_ \_\_\_\_ \_\_\_\_ \_\_\_\_ (256, 512, 1024, 2048, 4096)  
(Rule: Double [multiply by 2] each time.)

This last pattern is how Nathaniel Bowditch figured out how pretty his sister Lizza had become:

*You know what? Every year since you were eight, you've gotten twice as pretty as the year before.*

*"Hmmm..." Lizza bit her tongue and rolled her eyes. "I'm nineteen now...eight from nineteen..."*

*She sighed. "You mathematician! I wish you could at least pay a compliment without arithmetic!"*

*Eight from nineteen is eleven. Twice as pretty every year...Goodness, I'm twenty-two times as pretty!*

*Nat laughed. You're two thousand and forty-eight times as pretty. Keep count on your fingers as I double one*

Eleven times: two---four---eight---sixteen---thirty-two---sixty-four---one hundred twenty-eight---two hundred fifty-six--- five hundred twelve---one thousand and twenty-four--- and two thousand and forty-eight!"

"And next year..." Lizza said.

"Four thousand and ninety-six times as pretty!..."

(Carry On, Mr. Bowditch, 76)

Some patterns are little more complicated. Let's try some that are a bit harder.

26, 23, 28, 25, 30, 27, \_\_\_\_\_ (32, 29, 34, 31)

(Rule:  $-3+5-3+5...$ )

2, 5, 11, 23, \_\_\_\_\_ (47, 95, 191)

(Rule:  $\times 2 + 1 \times 2 + 1...$ )

32, 16, 8, 4, 2, \_\_\_\_\_ Be careful on this one! ( $1, 1/2, 1/4$ )

(Rule: Halve it [divide by 2] each time.)

$1/2, 2/3, 3/4, 4/5, \_\_\_\_\_\_$  ( $5/6, 6/7, 7/8$ )

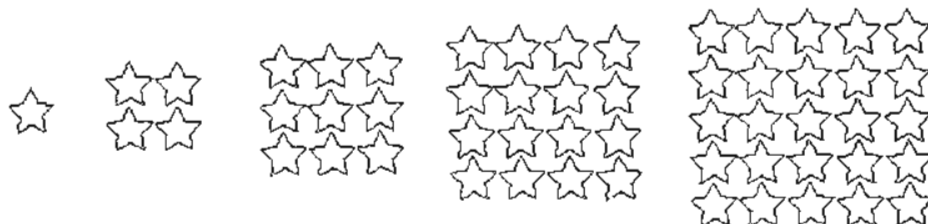
(Rule: Increase numerator by 1 and denominator by 1 each time.)

Some patterns are made up of special numbers. Try these:

1, 4, 9, 16, 25, \_\_\_\_\_ (36, 49, 64)

(Rule:  $1 \times 1, 2 \times 2, 3 \times 3, 4 \times 4, 5 \times 5, \dots$  or,  $+3+5+7+9...$ )

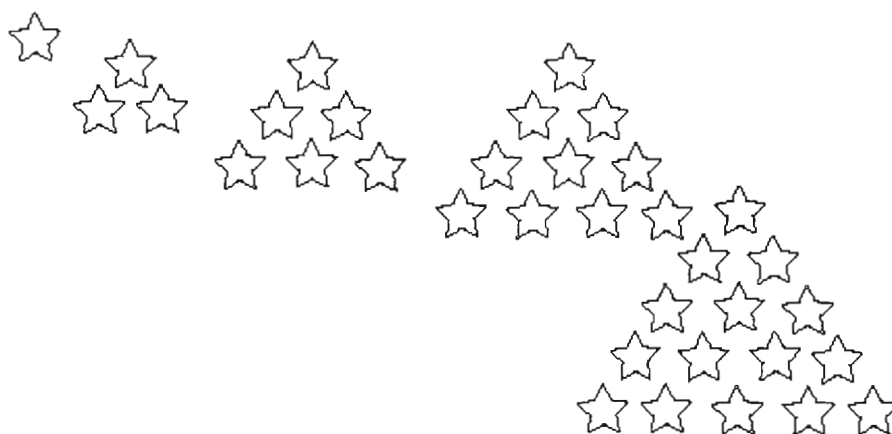
These numbers are called square numbers, because they can be represented by a square formation.



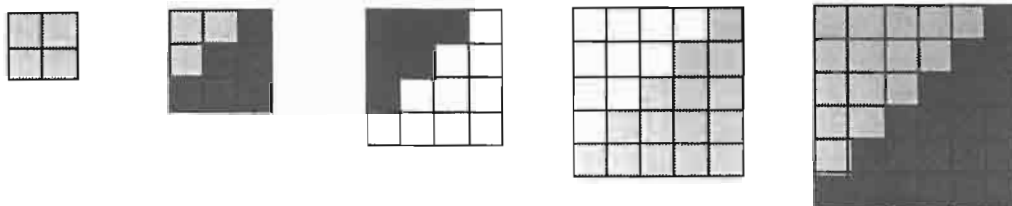
1, 3, 6, 10, 15, \_\_\_\_\_ (21, 28, 36)

(Rule:  $+2+3+4+5...$ )

These numbers are called triangular numbers, because they can be represented by a formation that looks like a triangle.



What happens when you add any two triangular numbers which are next to each other?



You get a square number. Give examples:

$$1+3=4$$

$$3+6=9$$

$$6+10=16$$

$$10+15=25$$

$$15+21=36$$

Boldface numbers are all square numbers.

The triangular and square numbers are related! Mathematics is full of relationships!

Try this one:

2, 6, 12, 20, 30, \_\_, \_\_, \_\_ (42, 56, 72)

(Rule: there are several rules that work here.  $+4+6+8+10\dots$  is one rule. Another rule that works is  $1\times 2, 2\times 3, 3\times 4, 4\times 5, 5\times 6, \dots$ . Still another rule is the observation that each number in the series is twice the corresponding triangular number above. All answers are acceptable. Encourage the creative thinking and the appreciation of the beauty in math!)

This next pattern has all sorts of rules governing it. Write the next three lines of the pattern. Then, as a class, discuss what patterns are going on within the pattern.

$$1 = 1 \times 1 = 1$$

$$9 = 3 \times 3 = 2+3+4$$

$$25 = 5 \times 5 = 3+4+5+6+7$$

$$49 = 7 \times 7 = 4+5+6+7+8+9+10$$

(Rule: The first column represents the squares of consecutive odd numbers.

Rule: The first number to the right of the second "=" in each row are the counting numbers

[consecutive positive integers].

Rule: The number of numbers added together equals the base number for the row.)

Now it's your turn. Pair off with a partner. Each of you make up a pattern and see if the other person can figure out the rule and predict the next three numbers.

## Using a Pattern to Solve a Problem More Quickly

If someone asked you to multiply 123456789 by 8 and then add 9 to the answer, how long do you think it would take you to compute the answer? Using patterns, you could find the answer much more quickly than you think! Study the pattern below, and then write the next five lines of the pattern. What is the answer to the problem above---

$123456789 \times 8 + 9$ ? Have someone check each line of the pattern with a calculator to make sure each equation is true.

$$1 \times 8 + 1 = 9$$

$$12 \times 8 + 2 = 98$$

$$123 \times 8 + 3 = 987$$

$$1234 \times 8 + 4 = 9876$$

Here is a similar pattern. You can finish it without any computation, but have someone check it with a calculator just to make sure.

$$1 \times 9 + 2 = 11$$

$$12 \times 9 + 3 = 111$$

$$123 \times 9 + 4 = 1111$$

Here is one more pattern for you to try. Again, complete the next five lines by following the pattern, and check your answers on a calculator.

$$9 \times 9 + 7 = 88$$

$$98 \times 9 + 6 = 888$$

$$987 \times 9 + 5 = 8888$$







## PATTERNS BEGIN SIMPLY

As you have probably noticed, patterns are everywhere---in a snowflake, a pine cone, a sunflower, on wallpaper, the kitchen floor, a quilt. Patterns are both beautiful and useful. What makes a pattern?

**Definition:** Pattern - a design or set of features that repeats in an orderly way or that has a rule to help you predict what will happen next.

Mathematics has patterns, too, and that makes mathematics predictable. You can figure out what will happen next and that can help you solve some difficult problems!

Let's look at a few simple mathematical patterns. Figure out a rule which might have been used to create each pattern and tell which numbers should go on the blanks.

1, 2, 3, 4, 5, \_\_\_\_

2, 4, 6, 8, \_\_\_\_

100, 95, 90, 85, 80, \_\_\_\_

1, 2, 4, 8, 16, 32, 64, 128, \_\_\_\_

This last pattern is how Nathaniel Bowditch figured out how pretty his sister Lizza had become:

*You know what? Every year since you were eight, you've gotten twice as pretty as the year before.*

*"Hmmm..." Lizza bit her tongue and rolled her eyes. "I'm nineteen now...eight from nineteen..."*

*She sighed. "You mathematician! I wish you could at least pay a compliment without arithmetic!"*

*Eight from nineteen is eleven. Twice as pretty every year...Goodness, I'm twenty-two times as pretty!*

*Nat laughed. You're two thousand and forty-eight times as pretty. Keep count on your fingers as I double one*

Eleven times: two---four---eight---sixteen---thirty-two---sixty-four---one hundred twenty-eight---two hundred fifty-six--- five hundred twelve---one thousand and twenty-four--- and two thousand and forty-eight!"

"And next year..." Lizza said.

"Four thousand and ninety-six times as pretty!..."

(*Carry On, Mr. Bowditch*, 76)

Some patterns are little more complicated. Let's try some that are a bit harder.

26, 23, 28, 25, 30, 27, \_\_, \_\_, \_\_, \_\_

2, 5, 11, 23, \_\_, \_\_, \_\_

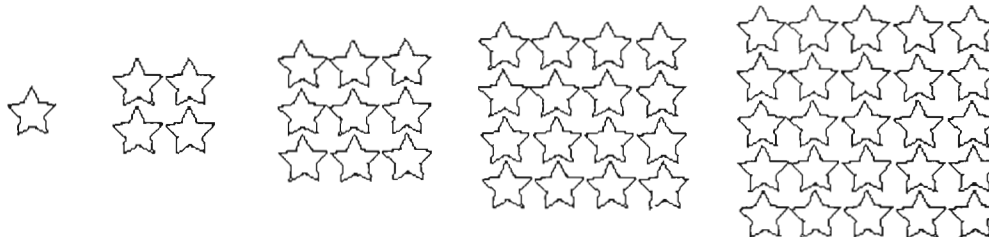
32, 16, 8, 4, 2, \_\_, \_\_, \_\_ Be careful on this one!

$1/2$ ,  $2/3$ ,  $3/4$ ,  $4/5$ , \_\_, \_\_, \_\_

Some patterns are made up of special numbers. Try these:

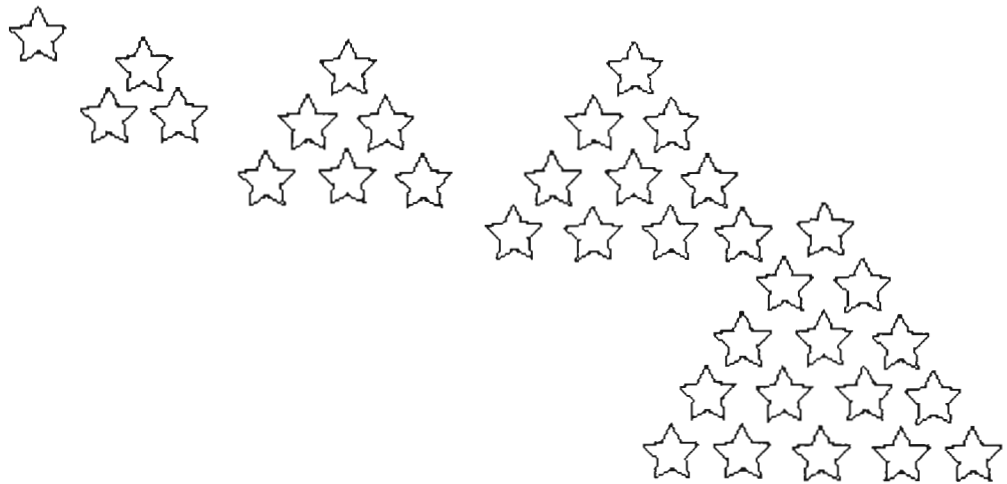
1, 4, 9, 16, 25, \_\_, \_\_, \_\_

These numbers are called square numbers, because they can be represented by a square formation.

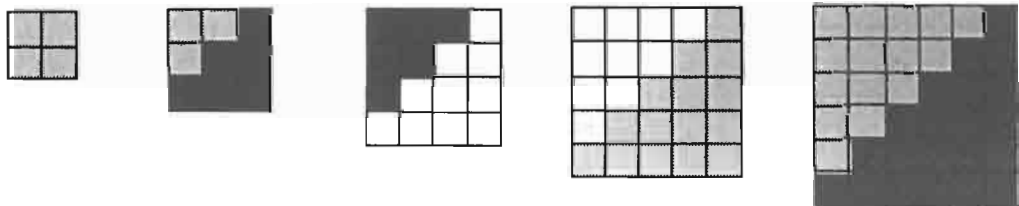


1, 3, 6, 10, 15, \_\_, \_\_, \_\_

These numbers are called triangular numbers, because they can be represented by a formation that looks like a triangle.



What happens when you add any two triangular numbers which are next to each other?



You get a square number. Give examples:

$$1+3=4$$

$$3+6=9$$

$$6+10=16$$

$$10+15=25$$

$$15+21=36$$

Boldface numbers are all square numbers.

The triangular and square numbers are related! Mathematics is full of relationships!

Try this one:

2, 6, 12, 20, 30, \_\_, \_\_, \_\_

This next pattern has all sorts of rules governing it. Write the next three lines of the pattern. Then, as a class, discuss what patterns are going on within the pattern.

$$1 = 1 \times 1 = 1$$

$$9 = 3 \times 3 = 2+3+4$$

$$25 = 5 \times 5 = 3+4+5+6+7$$

$$49 = 7 \times 7 = 4+5+6+7+8+9+10$$

Now it's your turn. Pair off with a partner. Each of you make up a pattern and see if the other person can figure out the rule and predict the next three numbers.



## USING A PATTERN TO SOLVE A PROBLEM MORE QUICKLY

If someone asked you to multiply 123456789 by 8 and then add 9 to the answer, how long do you think it would take you to compute the answer? Using patterns, you could find the answer much more quickly than you think! Study the pattern below, and then write the next five lines of the pattern. What is the answer to the problem above---

$123456789 \times 8 + 9$ ? Have someone check each line of the pattern with a calculator to make sure each equation is true.

$$1 \times 8 + 1 =$$

$$12 \times 8 + 2 =$$

$$23 \times 8 + 3 =$$

$$1234 \times 8 + 4 =$$

Here is a similar pattern. You can finish it without any computation, but have someone check it with a calculator just to make sure.

$$1 \times 9 + 2 =$$

$$12 \times 9 + 3 =$$

$$123 \times 9 + 4 =$$

Here is one more pattern for you to try. Again, complete the next five lines by following the pattern, and check your answers on a calculator.

$$9 \times 9 + 7 =$$

$$98 \times 9 + 6 =$$

$$987 \times 9 + 5 =$$





## Lesson 2: More Power to You

### Objectives:

- Students will complete tables by writing the first ten powers of 2, 3, 10, 11.
- Students will complete another table by showing the first ten cube numbers.
- Students will make observations and complete a table that shows the interrelationship among the cubes, the squares and the triangular numbers.

### Skills:

- Students will use simple exponential notation to express numbers in a pattern generated by powers.
- Students will use tables as a tool to organize data and look for meaningful patterns in the data.

### Vocabulary:

- exponent
- power
- cube number

### Materials:

- pencils, calculator
- 100-200 cubes of uniform size (dice will do)

### Procedure:

1. Distribute worksheet "More Power to You".
2. Show students how exponents provide an abbreviated way of multiplying the same factor repeatedly.
3. Complete the tables as a class.
4. As an optional activity, have students use blocks or dice to represent cube numbers visually.

### Handouts:

- Worksheet: "More Power to You" (3 pp.)

## More Power To You

Exponents are a shorthand way to calculate a number used repeatedly as a factor.

In the expression  $2^3$  (read 2 to the third power), 3 is an exponent, and 2 is the base. This expression,  $2^3$  means  $2 \times 2 \times 2$ , which is equal to 8. The 2 is used as a factor 3 times. The expression  $2^5$  (2 to the fifth power) means  $2 \times 2 \times 2 \times 2 \times 2$ , which is equal to 32. Here, 2 is used as a factor 5 times. Using this definition of exponent, complete the table below. A few examples are done to get you started.

$2^0$	$2^1$	$2^2$	$2^3$	$2^4$	$2^5$	$2^6$	$2^7$	$2^8$	$2^9$	$2^{10}$	$2^{11}$	$2^{12}$
1	2	4	8	16	32	64	128	256	512	1024	2048	4096

Do you recognize a pattern in the bottom row of the table? (The numbers double.)

In *Carry On, Mr. Bowditch*, Nathaniel Bowditch told his sister Lizza, "Tear this note across twelve times and scatter the four thousand and ninety-six pieces to the wind!" Notice from the table that  $2^{12}$  equals 4096.

How many pieces of paper would you get if you tore a piece of paper 6 times? ( $64=2^6$ )

Hold a piece of scrap paper in your hands. Tear it 0 times. How many pieces of paper do you have? Still 1, of course! Do you notice from the table that  $2^0=1$ ?

Now tear the paper once. How many pieces do you have now? You have 2, and  $2^1=2$ . The exponent tells you how many times to tear the paper, and the number below it in the table tells you how many pieces of paper you will end up with.

After tearing a paper 2 times, you have  $2^2 = \underline{\hspace{1cm}}$  pieces of paper.

After tearing a paper 3 times, you have  $2^3 = \underline{\hspace{1cm}}$  pieces of paper.

After tearing a paper 4 times, you have  $2^4 = \underline{\hspace{1cm}}$  pieces of paper.

In performing this little experiment, you have used two important problem-solving strategies:

- finding a pattern
- making a table

Use these strategies as you watch for patterns and fill in the tables below.

Remember:  $3^4 = 3 \times 3 \times 3 \times 3$ ,  $11^5 = 11 \times 11 \times 11 \times 11 \times 11$ , and so on. More power to you!



$3^0$	$3^1$	$3^2$	$3^3$	$3^4$	$3^5$	$3^6$
1	3	9	27	81	243	729

$10^0$	$10^1$	$10^2$	$10^3$	$10^4$	$10^5$	$10^6$
1	10	100	1000	10,000	100,000	1,000,000

$11^0$	$11^1$	$11^2$	$11^3$	$11^4$	$11^5$	$11^6$
1	11	121	1331	14641	161051	177,1561

Any number to the 0 power = 1? Why? (Because it fits the pattern.)

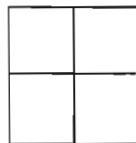
Any number to the first power (exponent = 1) = itself?

Notice the middle table, where 10 is the base. Did you see that the exponent also tells you the number of zeros to write after the 1? Therefore, what is  $10^9$ ? (1,000,000,000 = 1 billion)

## A Powerful Pattern

$1^3$	$2^3$	$3^3$	$4^3$	$5^3$	$6^3$	$7^3$	$8^3$	$9^3$	$10^3$
1	8	27	64	125	216	343	512	729	1000

The numbers in the bottom row of the table form a pattern because they are all cubes. Numbers to the third power are called cubes, because if you can form a cube shape with a cubed number of blocks. Take out 8 blocks from your kit. Make a square layer that is 2 blocks wide on each side like this:



Now place an identical second layer on top of the first. You have used 8 blocks to form a cube that is 2 blocks long, 2 blocks wide and 2 blocks high. That is why 8 is called  $2^3$  (read "2 cubed").

Now take 27 blocks and show why 27 is 3 cubed.

Here is a pattern where the cubes, the squares and the triangular numbers are all related!

Remember:

**Triangular Numbers:** 1, 3, 6, 10, 15, 21, 28, ...

**Squares:** 1, 4, 9, 16, 25, 36, 49, ...

**Cubes:** 1, 8, 27, 64, 125, 216, 343, ...

Let  $N$  = the number. Let  $S$  = the sum of the first  $N$  cubes. Let  $T$  = the  $n$ th triangular number. Let  $T^2$  = the square of  $T$ .

N	$S = 1^3 + 2^3 + 3^3 + \dots + N^3$	T	$T^2$
1			
2			
3	$S = 1^3 + 2^3 + 3^3 = 3^6$	6	36
4			
5			
6			
7	$S = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 = 784$	28	784
8			
9			
10			

Let's do a few examples together. Let  $N = 3$ .  $S = 1^3 + 2^3 + 3^3 = 3^6$ . The 3rd triangular number is 6. Therefore,  $T = 6$ .  $T^2 = 36$ . Notice that column 2 and column 4 are equal! This will happen for all the examples.

Let's do one more row together. Let  $N = 7$ .  $S = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 = 784$ .  $T$  = the 7th triangular number = 28.  $28^2 = 784$ . Again, and as predicted, the 2nd and 4th columns are the same.

Now, it's your turn. Complete the rest of the table and see how the cubes, the squares and the triangular numbers are all related.



## MORE POWER TO YOU

Exponents are a shorthand way to calculate a number used repeatedly as a factor.

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$2^0$	$2^1$	$2^2$	$2^3$	$2^4$	$2^5$	$2^6$	$2^7$	$2^8$	$2^9$	$2^{10}$	$2^{11}$	$2^{12}$
1	2	4	8	16	32	64						

Do you recognize a pattern in the bottom row of the table?

In *Carry On, Mr. Bowditch*, Nathaniel Bowditch told his sister Lizza, "Tear this note across twelve times and scatter the four thousand and ninety-six pieces to the wind!"

How many pieces of paper would you get if your tore a piece of paper 6 times?

Hold a piece of scrap paper in your hands. Tear it 0 times. How many pieces of paper do you have? Still 1, of course! Do you notice from the table that  $2^0=1$ ?

Now tear the paper once. How many pieces do you have now? You have 2, and  $2^1=2$ . The exponent tells you how many times to tear the paper, and the number below it in the table tells you how many pieces of paper you will end up with.

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Remember:  $3^4 = 3 \times 3 \times 3 \times 3$ ,  $11^5 = 11 \times 11 \times 11 \times 11 \times 11$ , and so on. More power to you!

$3^0$	$3^1$	$3^2$	$3^3$	$3^4$	$3^5$	$3^6$
1	3		27			

$10^0$	$10^1$	$10^2$	$10^3$	$10^4$	10	10
1	10	100	1000	10,000		

$11^0$	$11^1$	$11^2$	$11^3$	$11^4$	$11^5$	$11^6$
1	11	121	1331	14641		

Any number to the 0 power = \_\_\_\_? Why?

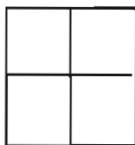
Any number to the first power (exponent = 1) = \_\_\_\_\_?

Notice the middle table, where 10 is the base. Did you see that the exponent also tells you the number of zeros to write after the 1? Therefore, what is  $10^9$ ?

## A Powerful Pattern

$1^3$	$2^3$	$3^3$	$4^3$	$5^3$	$6^3$	$7^3$	$8^3$	$9^3$	$10^3$
1	8	27	64	125	216	343	512	729	1000

The numbers in the bottom row of the table form a pattern because they are all **cubes**. Numbers to the third power are called **cubes**, because if you can form a **cube** shape with a **cubed** number of blocks. Take out 8 blocks from your kit. Make a square layer that is 2 blocks wide on each side like this:



Now place an identical second layer on top of the first. You have used 8 blocks to form a cube that is 2 blocks long, 2 blocks wide and 2 blocks high. That is why 8 is called  $2^3$  (read "2 cubed").

Now take 27 blocks and show why 27 is 3 cubed.

Here is a pattern where the cubes, the squares and the triangular numbers are all related!

Remember:

**Triangular Numbers:** 1, 3, 6, 10, 15, 21, 28, ...

**Squares:** 1, 4, 9, 16, 25, 36, 49, ...

**Cubes:** 1, 8, 27, 64, 125, 216, 343, ...

Let  $N$  = the number. Let  $S$  = the sum of the first  $N$  cubes. Let  $T$  = the  $n$ th triangular number. Let  $T^2$  = the square of  $T$ .

$N$	$S = 1^3 + 2^3 + 3^3 + \dots + N^3$	$T$	$T^2$
1			
2			
3	$S = 1^3 + 2^3 + 3^3 = 36$	6	36
4			
5			
6			
7	$S = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 = 784$	28	784
8			
9			
10			

Let's do a few examples together. Let  $N = 3$ .  $S = 1^3 + 2^3 + 3^3 = 36$ . The 3rd triangular number is 6. Therefore,  $T = 6$ .  $T^2 = 36$ . Notice that column 2 and column 4 are equal! This will happen for all the examples.

Let's do one more row together. Let  $N = 7$ .  $S = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 = 784$ .  $T$  = the 7th triangular number = 28.  $28^2 = 784$ . Again, and as predicted, the 2nd and 4th columns are the same.

Now, it's your turn. Complete the rest of the table and see how the cubes, the squares and the triangular numbers are all related.





## Lesson 3: A Very Old Problem

### Objectives:

- Students will work together to answer an old brain teaser:

How many squares on a checkerboard?

### Skills:

- Students will learn to use three problem-solving strategies to solve this riddle:
  - a) solving a simpler problem
  - b) making a table to record data
  - c) observing a pattern in the data

### Vocabulary:

- problem-solving strategies

### Materials:

- pencils, acetate squares, grid overlays

### Procedure:

1. Ask how many squares on a checkerboard?

(The answer is not '64'.)

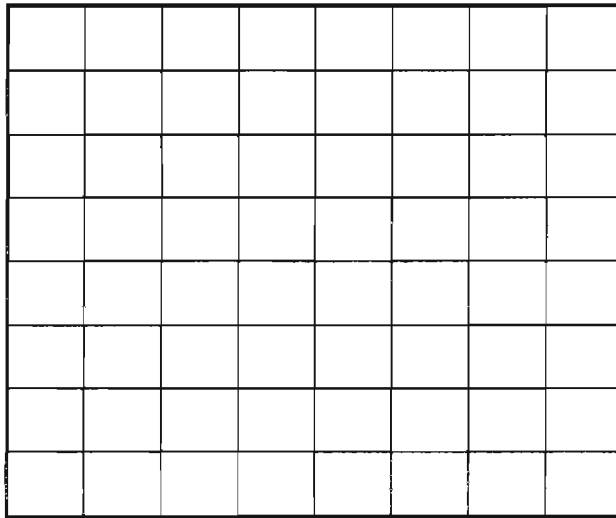
2. Distribute and proceed with worksheet "A Very Old Problem".
3. Use acetate squares and grid overlays to count the various sizes of the squares.

### Handout:

- Worksheet: "A Very Old Problem" (3 pp.)

## A Very Old Problem

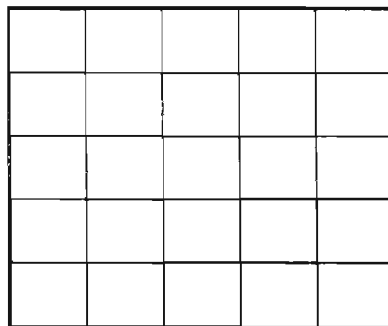
Here is a very old brain teaser. How many squares on a checkerboard?



If you said 64, you're not done yet, because you did not count all the squares. We're going to use three problem-solving strategies you have learned to solve this problem.

- 1) We will solve a simpler problem.
- 2) We will make a table.
- 3) We will look for a pattern.

First, let's solve a similar, but simpler, problem. We'll find out how many squares in a 5-by-5 square grid instead of an 8-by-8 square grid.



For the following section, please use red, blue, green, gold and purple acetate squares:



We have to look at the different size squares. The whole 5-by-5 square checkerboard (outline in blue) is a square.

A	B	C	D	E
F	G	H	I	J
K	L	M	N	O
P	Q	R	S	T
U	V	W	X	Y

Then, we have some 4-by-4 squares (outline in red, green, gold, purple). There are four of those 4-by-4 squares. We have, of course, the twenty-five individual 1-by-1 squares, but we need to figure out how many 2-by-2 squares and 3-by-3 squares we have.

Before we proceed, let's make a table to record the data we already have.

SQUARE SIZE	HOW MANY?
1-by-1 squares	25
2-by-2 squares	16
3-by-3 squares	9
4-by-4 squares	4
5-by-5 squares	1

Now, we'll count the 3-by-3 squares. You may use the red acetate square in your kit to help you with this. Cover cells A, B, C, F, G, H, K, L, M. That's one 3-by-3 square. Take the red square and cover cells B, C, D, G, H, I, L, M, N. That's another 3-by-3 square. Move the red square around to find the other 3-by-3 squares:

C, D, E, G, H, I, L, M, N

F, G, H, K, L, M, P, Q, R

G, H, I, L, M, N, Q, R, S

H, I, J, M, N, O, R, S, T

K, L, M, P, Q, R, U, V, W

L, M, N, Q, R, S, V, W, X

M, N, O, R, S, T, W, X, Y

That makes a total of nine 3-by-3 squares. Write 9 in the second column of the table next to 3-by-3 squares. Take out the green acetate square from your kit and use it to find all the 2-by-2 squares.

*(Teachers: encourage student to be systematic—i.e., orderly---not haphazard or random in looking for the squares.)*

Did you find sixteen 2-by-2 squares? Write 16 in the second column of the table next to 2-by-2 squares. Starting at the bottom of the second column in the table and reading upward, do you recognize a familiar pattern?

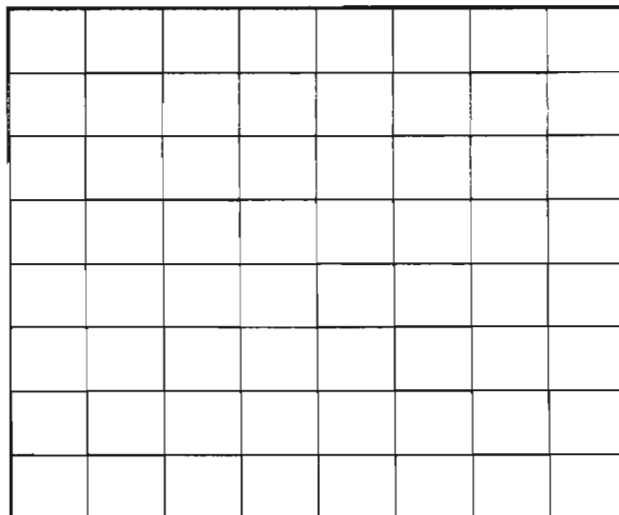
*(square numbers)*

Add up all the numbers in the second column of the table to get the total number of squares in a 5-by-5 square grid. (55)

Now we can go back to the original problem: how many squares in a checkerboard (8-by-8 square grid)?

We've already solved a simpler problem so we'll use the other two problem-solving techniques we just used---

- We will make a table.
- We will look for a pattern.



Here is the table to help you organize your data:

SQUARE SIZE	HOW MANY?
1-by-1 squares	64
2-by-2 squares	49
3-by-3 squares	36
4-by-4 squares	25
5-by-5 squares	16
6-by-6 squares	9
7-by-7 squares	4
8-by-8 squares	1

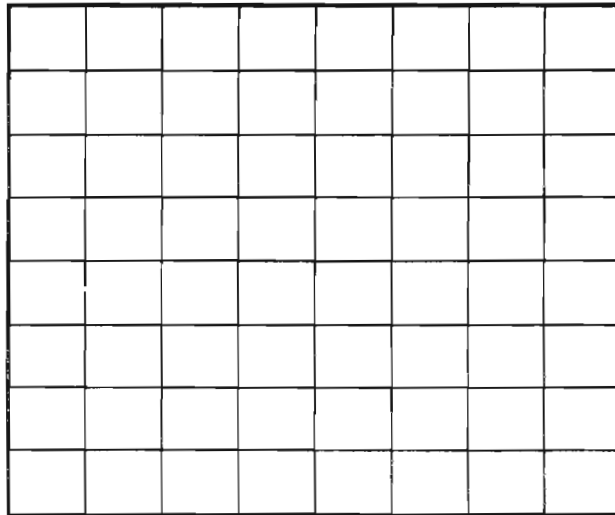
Hint: Count the number of 1-squares, 2-squares, 7-squares and 8-squares, and see if you notice a pattern forming. If you recognize it, you'll have this problem solved in no time at all! (Total=204)





## A VERY OLD PROBLEM

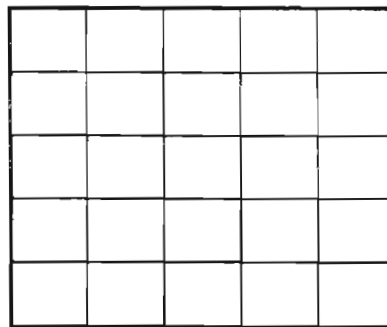
Here is a very old brain teaser. How many squares on a checkerboard?



If you said 64, you're not done yet, because you did not count all the squares. We're going to use three problem-solving strategies you have learned to solve this problem.

- 1) We will solve a simpler problem.
- 2) We will make a table.
- 3) We will look for a pattern.

First, let's solve a similar, but simpler, problem. We'll find out how many squares in a 5-by-5 square grid instead of an 8-by-8 square grid.



For the following section, please use red, blue, green, gold and purple acetate squares:

We have to look at the different size squares. The whole 5-by-5 square checkerboard (outline in blue) is a square.

A	B	C	D	E
F	G	H	I	J
K	L	M	N	O
P	Q	R	S	T
U	V	W	X	Y

Then, we have some 4-by-4 squares (outline in red, green, gold, purple). There are four of those 4-by-4 squares. We have, of course, the twenty-five individual 1-by-1 squares, but we need to figure out how many 2-by-2 squares and 3-by-3 squares we have.

Before we proceed, let's make a table to record the data we already have.

SQUARE SIZE	HOW MANY?
1-by-1 squares	
2-by-2 squares	
3-by-3 squares	
4-by-4 squares	4
5-by-5 squares	1

Now, we'll count the 3-by-3 squares. You may use the red acetate square in your kit to help you with this. Cover cells A, B, C, F, G, H, K, L, M. That's one 3-by-3 square. Take the red square and cover cells B, C, D, G, H, I, L, M, N. That's another 3-by-3 square. Move the red square around to find the other 3-by-3 squares:

C, D, E, G, H, I, L, M, N  
 F, G, H, K, L, M, P, Q, R  
 G, H, I, L, M, N, Q, R, S  
 H, I, J, M, N, O, R, S, T  
 K, L, M, P, Q, R, U, V, W  
 L, M, N, Q, R, S, V, W, X

M, N, O, R, S, T, W, X, Y

That makes a total of nine 3-by-3 squares. Write 9 in the second column of the table next to 3-by-3 squares. Take out the green acetate square from your kit and use it to find all the 2-by-2 squares.

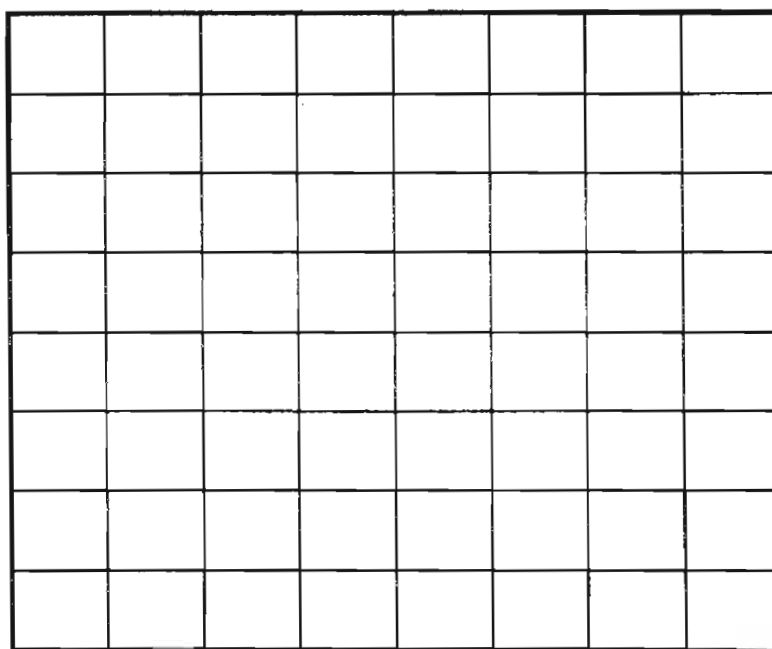
Did you find sixteen 2-by-2 squares? Write 16 in the second column of the table next to 2-by-2 squares. Starting at the bottom of the second column in the table and reading upward, do you recognize a familiar pattern?

Add up all the numbers in the second column of the table to get the total number of squares in a 5-by-5 square grid.

Now we can go back to the original problem: how many squares in a checkerboard?

We've already solved a simpler problem so we'll use the other two problem-solving techniques we just used---

- We will make a table.
- We will look for a pattern.



Here is the table to help you organize your data:

SQUARE SIZE	HOW MANY?
1-by-1 squares	64
2-by-2 squares	49
3-by-3 squares	
4-by-4 squares	
5-by-5 squares	
6-by-6 squares	
7-by-7 squares	4
8-by-8 squares	1

Hint: Count the number of 1-squares, 2-squares, 7-squares and 8-squares, and see if you notice a pattern forming. If you recognize it, you'll have this problem solved in no time at all!





## Lesson 4: Design-a-Quilt

### Objectives:

- Students will design a quilt pattern within a 12 block-by-12 block square on graph paper in such a way that the design has regularity with respect to all four sides of the square.

### Skills:

- Students will be able to count squares on graph paper to help measure and proportion their designs.

### Vocabulary:

- Proportion

### Materials:

- graph paper
- ruler, pencils
- colored markers
- compass (optional)

### Procedure:

1. Give each student a piece of graph paper, and have him/her use a pencil and ruler to draw a square 12 blocks by 12 blocks.

(24 by 24 works well also).

2. These numbers are divisible by 2,3, 4, 6, and most of the designs are based on these numbers. For example, designs B and E are based on 3, since the foundation of the design started out with  $9 \div 3 = 3$  major squares.
2. Use a book of quilt designs as inspiration and have each student choose a design. Each student will construct a design that looks the same every time the design is rotated  $90^\circ$ .
3. Encourage students who want more of a challenge to try a design with curves. (They will need to figure out where the center of the circle would be if the curve had become a complete circle.)

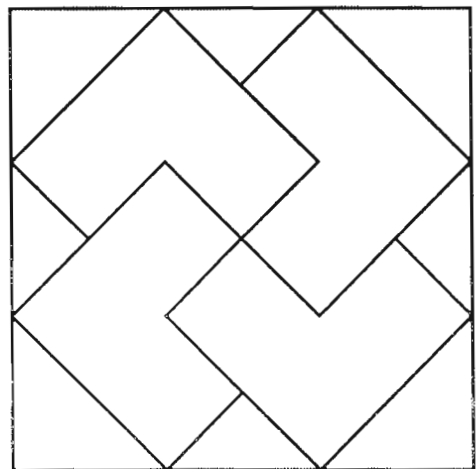
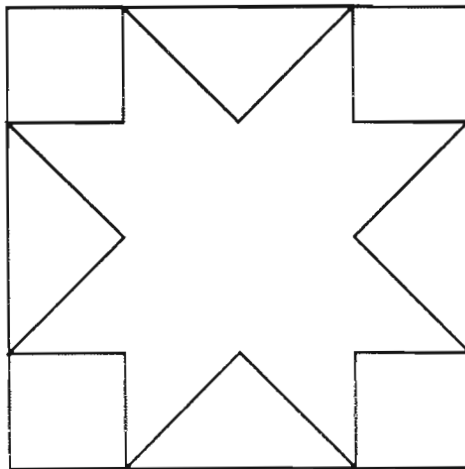
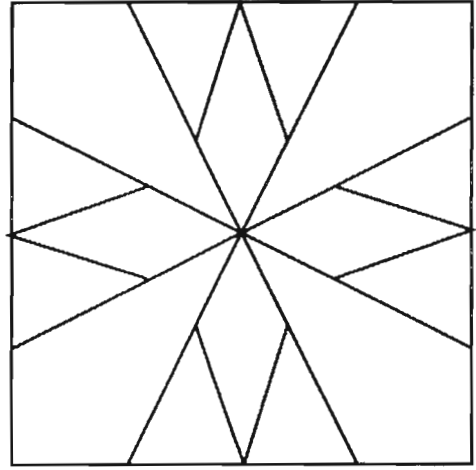
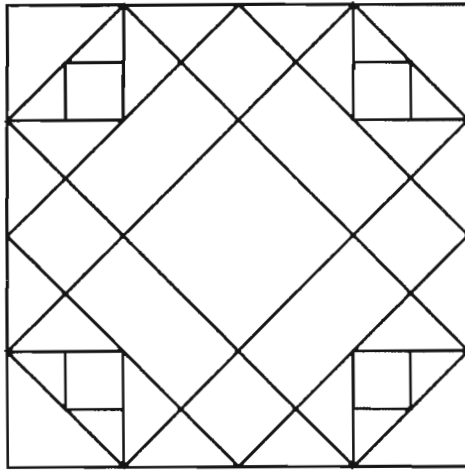
### Handouts:

- Quilt Blocks (3 pp.)





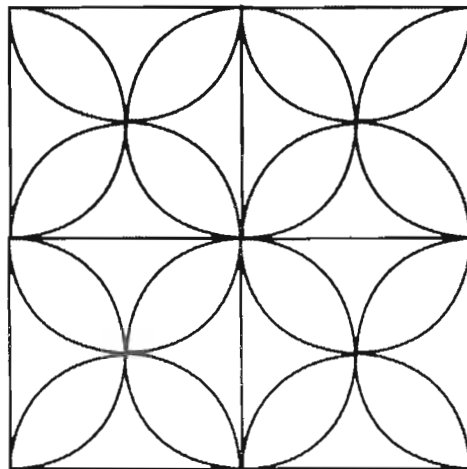
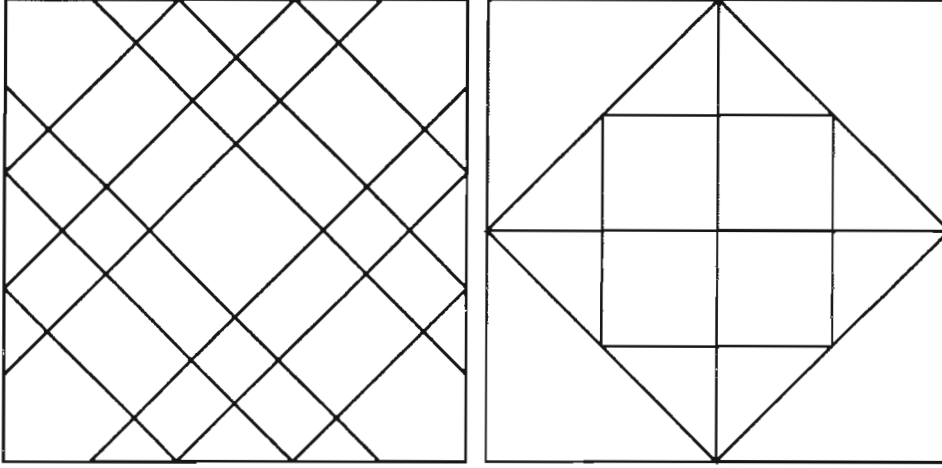
# QUILT BLOCK HANDOUT 1



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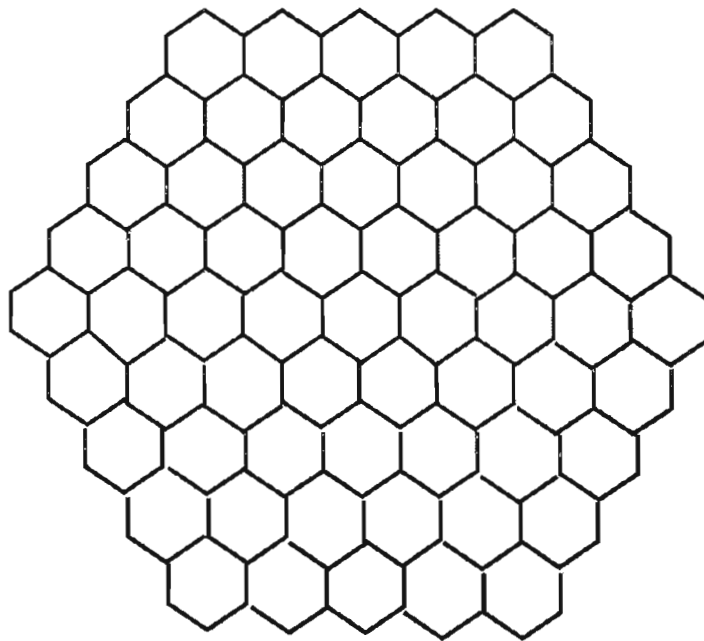
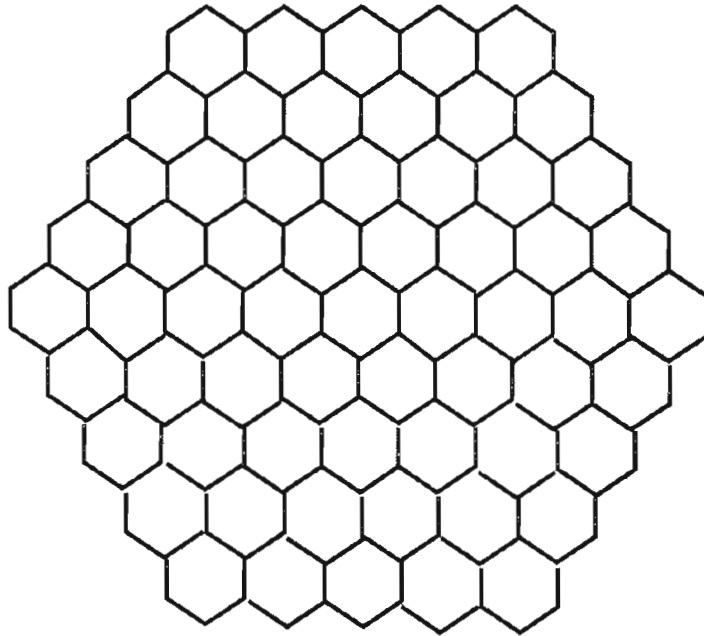
## QUILT BLOCK HANDOUT 2



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# QUILT BLOCK HANDOUT 3



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## Lesson 5: Fibonacci Fun

### Objectives:

- Students will make observations about this very special pattern that has been known since ancient times, but made famous by Leonardo da Fibonacci ca.1200 C.E.
- Students will name examples of Fibonacci numbers in nature and in aesthetics.

### Skills:

- Students will learn to look for patterns, as well as patterns within the patterns in the Fibonacci sequence.

### Vocabulary:

- Consecutive

### Materials:

- optional: sunflower, pine cones, pineapple

### Procedure:

1. Distribute worksheet "Fibonacci Fun."
2. Do mathematical activities A, B, C together as a class
3. Discuss leaf arrangements, pine cone, pineapple and sunflower spirals as they relate to Fibonacci numbers.
4. Have students look at the pairs of rectangles on the third page of handout and 'vote' for the one in each pair they like best. X2, Y1, Z1 are the Fibonacci rectangles and will likely appeal to most people. These rectangles can be drawn on a chalk board: 7x14, 5x8; 3x5, 12x3; 13x8, 10x10.

### Handout:

- Worksheet "Fibonacci Fun" (3 pp.)

## Fibonacci Fun

In 1202, a mathematician named Leonarda da Pisa (a.k.a. Fibonacci) published a sequence of numbers which we now call the Fibonacci numbers. This sequence, or pattern, of numbers starts out like this:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...

The three dots mean the series (like most series) goes on forever. Write down the next eight Fibonacci numbers:

89, 144, 233, 377, 610, 987, 1597, 2584

Fibonacci numbers have many interesting mathematical properties. Here are some activities for you to try.

A. Take any three consecutive Fibonacci numbers. (Consecutive means in a row' or 'one right after the other.') Let's choose 5, 8, 13. Square (i.e., multiply it by itself) the middle number.  $8 \times 8 = 64$ . Take the first and last numbers and multiply them together.  $5 \times 13 = 65$ . How does this product compare with the squared number? (The difference is always 1.) Now, choose three different consecutive Fibonacci numbers and do this activity with the new numbers. Did you get a difference of 1 again?

B. Square each term of the Fibonacci series. Let's just start with the first five Fibonacci numbers: 1, 1, 2, 3, 5. Now we'll square each of these numbers

$1 \times 1, 1 \times 1, 2 \times 2, 3 \times 3, 5 \times 5$

and write down the new series:

1, 1, 4, 9, 25

Add each consecutive pair of numbers in the new series:

$1+1, 1+4, 4+9, 9+25$

Write down the newest series:

2, 5, 13, 34

What do you have? (all Fibonacci numbers!)

Do this same exercise with the next five Fibonacci numbers. (8, 13, 21, 34, 55)

C. Choose any three consecutive Fibonacci numbers. Let's use 5, 8, 13 again as an example. Cube each of the two larger numbers.  $13 \times 13 \times 13 = 2197$  and  $8 \times 8 \times 8 = 512$ . Add these cubes together:  $2197 + 512 = 2709$ . Subtract



the cube of the smallest number.  $5 \text{ cubed} = 5 \times 5 \times 5 = 125$ .  $2709 - 125 = 2584$ . The result is a Fibonacci number!

Besides all these amazing mathematical properties (and there are many more), Fibonacci numbers pop up all over nature. Many flowers have a Fibonacci number of petals. Enchanter's Nightshades have 2 petals, lilies and irises have 3 petals, wild geraniums have 5 petals, delphinium have 8 petals, corn marigolds have 13 petals, chicory and aster have 21 petals, ox-eye daisies have 34 petals, field daisies have 55 petals, and so on.

In some trees and plants, leaves spiral around the stems in Fibonacci patterns. The number of turns required to find a leaf directly above another leaf is a Fibonacci number and the number of leaves from one leaf to another directly above it is a Fibonacci number.

See illustrations below.

TREE NAME	NO. OF TURNS	NO. OF LEAVES
beech	1	3
apricot	2	5
pear	3	8
almond	5	13

If you look at the seeds of a sunflower, you will notice a set of clockwise spirals and a set of counterclockwise spirals. There might be 21 spirals going one way and 34 spirals going the other way, or 34 spirals one way and 55 spirals the other, but always a pair of consecutive Fibonacci numbers.

The same is true of the spirals made by pine cone scales and pineapple scales. Pine cones usually have 5 spirals winding one way and 8 winding the other way; pineapples usually show 8 spirals one way and 13 spirals the other way.

Teachers: Have the students look at the pairs of rectangles on the next page. For each pair, have students "vote" on which each pair is most pleasing to the eye. The majority will most likely vote for the rectangles with the Fibonacci proportions: X1 ( $5 \times 8$ ),

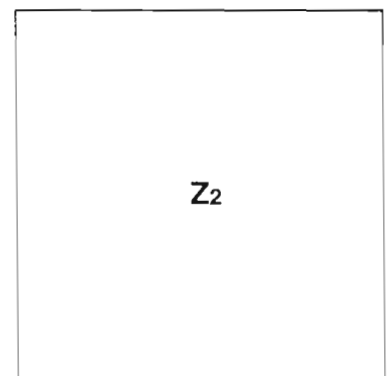
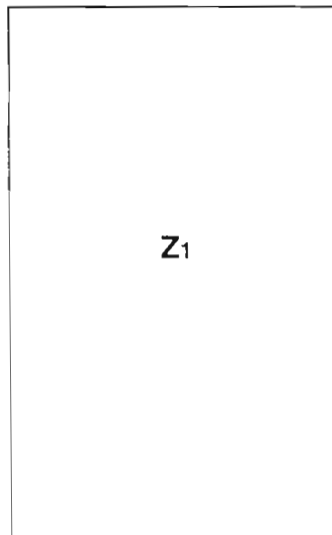
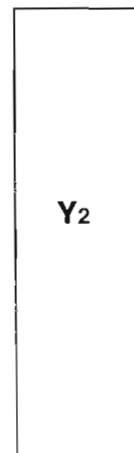
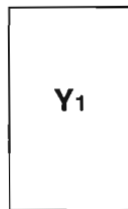
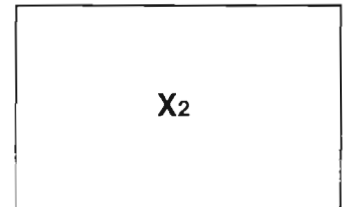
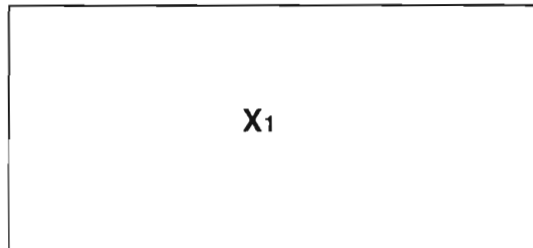
Y2 ( $3 \times 5$ ), Z1 ( $8 \times 13$ )

Which of each pair of rectangles is more appealing to you?





# WHICH OF EACH PAIR OF RECTANGLES IS MORE APPEALING TO YOU?





## FIBONACCI FUN

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The three dots mean the series (like most series) goes on forever. Write down the next eight Fibonacci numbers:

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B. Square each term of the Fibonacci series. Let's just start with the first five Fibonacci numbers: 1, 1, 2, 3, 5. Now we'll square each of these numbers

$1 \times 1, 1 \times 1, 2 \times 2, 3 \times 3, 5 \times 5$

and write down the new series:

Add each consecutive pair of numbers in the new series:

Write down the newest series:

What do you have?

Do this same exercise with the next five Fibonacci numbers.

C. Choose any three consecutive Fibonacci numbers. Let's use 5, 8, 13 again as an example. Cube each of the two larger numbers.  $13 \times 13 \times 13 =$  and  $8 \times 8 \times 8 =$  . Add these cubes together: Subtract the cube of the smallest number.  $5 \text{ cubed} = 5 \times 5 \times 5 =$  . The result is a Fibonacci number!

Besides all these amazing mathematical properties (and there are many more), Fibonacci numbers pop up all over nature. Many flowers have a Fibonacci number of petals. Enchanter's Nightshades have 2 petals, lilies and irises have 3 petals, wild geraniums have 5 petals, delphinium have 8 petals, corn marigolds have 13 petals, chicory and aster have 21 petals, ox-eye daisies have 34 petals, field daisies have 55 petals, and so on.

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See illustrations below.

TREE NAME	NO. OF TURNS	NO. OF LEAVES
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If

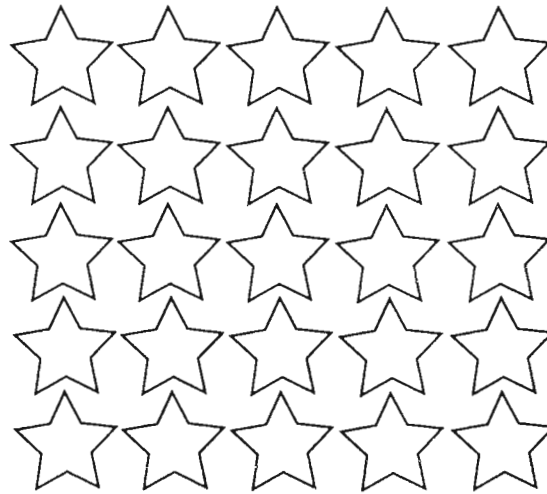
you look at the seeds of a sunflower, you will notice a set of clockwise spirals and a set of counterclockwise spirals. There might be 21 spirals going one way and 34 spirals going the other way, or 34 spirals one way and 55 spirals the other, but always a pair of consecutive Fibonacci numbers.

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## Unit 4:

# PATTERNS IN ASTRONOMY STARS



## OVERVIEW FOR TEACHERS

### Unit Outline

#### Introduction

...Ptolemy once wrote: 'in studying the convoluted orbits of the stars my feet do not touch the earth, and seated at the table of Zeus himself, I am nurtured with celestial ambrosia.'

– Anthony Aveni, *Stairways to the Stars : Skywatching in Three Great Ancient Cultures*. p. 193

Nathaniel Bowditch thrived on discovering meaningful patterns in numbers and simplifying complex phenomena for ordinary people. As a young boy in the chandlery, he was well aware that Salem's worldly commerce and wealth were intertwined with the mysterious study of navigation and the science of astronomy. Throughout his career, astronomy was Nathaniel's passion. Like Sir Isaac Newton before him, Nathaniel considered the moving heavens to be a never-ending challenge to arrange, describe, classify, and above all, to mathematically order. In his eyes, the seemingly mechanical movements of the stars and planets could be explained with precision and clarity.

Nathaniel's approach to learning and teaching, as Henry David Thoreau would later subscribe to, was to "Simplify, simply."

Nathaniel Bowditch was inspired early on to study the stars from walks on dark nights with his mother, observing the soothing affect of the heavens:

*She (Nathaniel's mother) and Nat went out into the dark, moonless night, and walked down Turner's Lane and out on the wharf. Mother helped Nat find the North Star, and told him how the Big Dipper swung around it, and how to tell time by the Dipper. Then she was silent, standing with her hand on Nat's shoulder, looking up at the stars. Boys don't blubber. He must remember that. Finally Nat said, "It's all right about school, Mother, when times are better, I'll get to go back.." Mother did not answer. She was still gazing up at the sky. After a while she said, "I made up a sort of saying for myself, Nat. I will lift up my eyes unto the stars. Sometimes, if you look at the stars long enough, it helps. It shrinks our day-by-day troubles down to size. (Carry on Mr. Bowditch. p. 33.)*

The Rev. Alexander Young later eulogized the life of Nathaniel Bowditch and relayed this early story about the moon and a sailor's wife's superstition.

*I found the plain two-story house, with but two rooms in it, where he dwelt with his mother; and I saw the chamber window where he said she used to sit and show him 'the new moon with the old moon under her arm,' and with the poetical superstition of a sailor's wife, jingle the silver in her pocket that her husband might have good luck, and she good tidings of him, far off upon the sea. (from Susan Bowditch, 1997, p. 6)*

The epitome of Bowditch's passion was the translation and expansion of the brilliant French astronomical text, *Mécanique Celeste*. This privately published American edition, titled *Translation and Commentary of Mécanique Celeste*, analyzed, corrected, and made readable the difficult scientific content of the original work. Bowditch's English language edition not only complemented the original work, but was considered by some as the best follow-up to Newton's *Principia*. Nathaniel's insistence on simplification and clarity allowed many more scientists the opportunity to understand Newton's laws of motion, particularly the role of gravity on the orbiting moon and planets.

Possibly Bowditch's most interesting talent was his aptness to teach difficult topics such as astronomy and navigation to the sailors on the Salem merchant vessels. Although he never accepted a formal teaching position, his many endeavors benefited from his insistence on clarity of instruction and presentation. For instance, in both his publications, *The American Practical Navigator* and *Translation and Commentary of Mécanique Celeste*, Bowditch added diagrams, text, and tables to clarify and better illustrate these difficult texts. Sailors on any one of his voyages benefited from his instruction and penchant for simplicity, as did scholars on both sides of the Atlantic. (Susan Bowditch)



The following unit, entitled Stars, parallels Nathaniel's lifelong pursuit of the very ancient science of astronomy. Using a planisphere in Lesson 1, students are challenged to find the rhythms in the sky by tracing the nightly movement of several stars and constellations. Circumpolar stars and constellations are then distinguished from seasonal stars and constellations. The movement of the stars along the ecliptic, the zodiac constellations, can also be discerned from the planisphere tracings and activities.

In Lesson 2, students explore myths. Ancient cultures memorized the nightly and seasonal sky patterns. Agricultural societies depended on these sky patterns to accurately predict the appropriate times for planting, harvesting, the season of flooding, etc. Fantastic stories of gods and goddesses in the night sky provided these cultures with effective memory aides to guide these predictions. Through explorations, students will discover that seasonal star groups are related to many Greek myths. In an independent research project, students are asked to adopt a constellation and to create a project describing their constellation and its characteristics.

Finally, in Lesson 3, students are led away from the movements of the stars and constellations to explore a more modern approach to star gazing. Bowditch's lifelong passion for astronomy and his insistence that complex patterns can be simplified is pushed into the twentieth century in an exercise using visible starlight data. In a segue activity to the study of visible light, students examine the light characteristics of our own Sun in relationship to other stars. Students sequentially organize and group data on starlight characteristics, such as temperature, absolute magnitude (brightness), luminosity, spectral type, and color, and then create special bar graphs, called histograms, to better understand these variables. As they compare and analyze this star data, students discover that our Sun is a medium-size, medium-temperature yellow star in a sea of blue, blue-white, white, yellow, and red stars of varying sizes, temperature and brightness.

Students continue to analyze graphs of absolute magnitude, temperature, color, luminosity, and spectral type. In their analyses, students uncover obvious similarities between star temperature and star color. Students also discover similarities between absolute magnitude and luminosity, and between spectral type and color. Finally, in a re-creation of a famous diagram in modern astronomy, the Hertzsprung-Russell diagram, students plot star temperature, absolute magnitude (brightness), and color on a single graph. This diagram, often called the workhorse of astronomy, provides scientists valuable insights into the nature of starlight. In addition to illustrating the important relationships between a star's brightness, heat, and color, the H-R diagram illustrates the natural life cycle or evolution of stars, including our Sun.

With their new appreciation of starlight, students are ready to examine the characteristics of light from our own Sun, using the early prism experiments of Sir Isaac Newton. Thus, our Nathaniel Bowditch theme returns to Nat's early mentor, Sir Isaac Newton, and Newton's development of the particle theory of light. [For more information on the nature of starlight and the creation of the Hertzsprung-Russell diagram, read on].

## The Nature of Starlight

Visible starlight data provides astronomers with invaluable information about the life and composition of each star. The feeble amount of light that reaches our Earth-based telescopes is analyzed in a spectroscope. The spectroscope splits light into its characteristic colors, or spectrum. This spectrum represents wavelengths of visible light ranging from violet blue (shortest wavelength) to red (longest wavelength). The resulting spectrogram is a series of black lines across a rainbow of colors, arranged right to left in the familiar mnemonic ROY G. BIV (red, orange, yellow, green blue, indigo, and violet) (Kerrod, 1993, p. 70).

Stars with a similar spectrum are classified into the same spectral classes, designated O, B, A, F, G, K, M, in order of decreasing temperature. The common mnemonic for these spectral classes is "Oh Be A Fine Girl (Guy) Kiss Me ". Each spectral class is associated with a peak color in the star spectrum and a defined range of temperatures:

O	B	A	F	G	K	M
Blue	Blue-White	White	Yellow-White	Yellow	Orange	Red

The classification of stars using spectral groups has an unusual history. Three women astronomers, all at the Harvard College Observatory at the end of the 1800's and early 1900's, played leading roles in developing the modern classification: Williamina Fleming, Antonia Maury, and Annie Jump Cannon. Most importantly, Annie Jump Cannon (1863-1941), a native of Nantucket, analyzed the spectral lines of 1,100 different stars. Her classification was based on rearranging the star spectra due to the presence of helium, nitrogen, and silicon. Her sequence O,B,A,F,G,K, and M had the effect of arranging stars according to color, ranging from blue (O) stars to red (M) stars. Modern astronomers now know that her spectral classes correspond to different star temperatures, with the hottest stars (blue) around 40,000 Kelvins (70,000° F). Her work was an important contribution to the later research of astronomer Ejnar Hertzsprung. (Trefil, 2000, p. 66; Burnham, Dyer, Garfinkle, George, Kanipe, Levy, 1997, p. 163).

Starlight reaching our planet is also measured according to its brightness, or magnitude. An instrument, called a photometer, can accurately measure a star's magnitude accurately to one or two decimals. A star's true brightness, or absolute magnitude, is the magnitude scientists would observe if a star were at a distance of 32.6 light-years. Apparent magnitude is the star's brightness as we see it from Earth, and thus has no relation to its true brightness. The scale for star absolute magnitude or brightness ranges from a -8 for the brightest star, to a +14 for the dimmest star. Our sun's absolute magnitude is 4.8, not a particularly bright star.

The brightness of each star is also termed star luminosity. Luminosity, or the total light energy emitted by a star, is measured either as absolute magnitude or in relation to the luminosity of our own Sun, which has a luminosity of one.

### **The Hertzsprung-Russell Diagram**

In 1911, a Dutch astronomer named Ejnar Hertzsprung (1873-1967) compared the spectra or color of stars with their corresponding luminosity. Two years later, unaware of Hertzsprung's work, an American astronomer, Henry Norris Russell (1877-1957) compared star absolute magnitude and spectral class. The spectral class of stars, of course, is based on the color spectrum. Luminosity and absolute magnitude both measure brightness. These two astronomers had independently discovered the same relationships in the stars unaware of each other's work. (Moore, 1986. P. 81).

The final diagram is now known as the Hertzsprung-Russell diagram, a workhorse in astronomy (Trefil, Nov, 2000, p. 67). The vertical axis represents star luminosity (in suns) and the horizontal axis represents star temperature (degrees Kelvin). This amazing diagram illustrates three important groupings of stars: main sequence stars, red giant stars, and white dwarf stars. Main sequence stars represent a broad band on the diagram running from the upper left (hot and bright) to the lower right (cool and dim). These stars, which include our Sun, generate their energy by fusing hydrogen into helium. Red giant stars (cool and bright), are a grouping in the upper right hand corner. The third group, white dwarfs (hot and dim) are in the lower left. These stars are cooling off, no longer producing the hydrogen to helium reaction of the main sequence stars.

The second remarkable feat produced by the H-R diagram is the representation of the life cycle of every star by a trajectory. James Trefil (Nov., 2000. p. 67), renowned science author, lecturer, and physicist, describes this evolutionary process using our sun, a main sequence star, as an example:

"The sun begins on the right [of the H-R diagram] as a cool, contracting cloud of interstellar gas. As it warms up, it moves left toward the main sequence. Finally, when the nuclear fires ignite and the star begins to fuse hydrogen, it sits on the main sequence, staying more or less in one place until all the hydrogen in the core is consumed. The sun will spend about 11 billion years on the main sequence, of which 4.6 billion years have already passed. When the hydrogen in the core is exhausted, more complex nuclear reactions begin. These reactions will cause the sun's surface to cool and swell up, at which point the sun moves to the red giant part of the diagram. Finally, when all nuclear reactions stop, the sun becomes a hot cinder in space and moves down to the white dwarf region."

Massive stars, or stars at their formation with at least six times the mass of our Sun, finish off their life in a different set of events. As they become red giants, massive stars do not continue on as white dwarfs, but continue to heat up fusing the carbon atoms into new heavier elements such as oxygen and nitrogen. The core of the massive star is so hot that fusion continues until the heavy element, iron, forms. At some point, the iron core begins to absorb energy. This energy is eventually released in a tremendous explosion called a supernova. The heat of the star can reach up to  $1,000,000,000^{\circ}\text{C}$ . The iron in the supernova fuse together to form new elements. The resulting gas cloud forms a new nebula, a baby star. The cycle of star birth to star death begins all over again. (Exploring Earth Science, Prentice Hall, 1997, pp. 81-84).

Stars with a beginning mass 1.5 to 4 times that of our sun will end up as a neutron star after a supernova. The mass of only one teaspoon of a neutron star would be about 100,000,000 tons. Pulsars are simply neutron stars that give off radio waves.

Finally, those stars with a mass 10 times that of our sun will end up as a black hole following the supernova explosion. The core of the star that remains after the supernova explosion is so massive and contains so little energy, it is eventually swallowed by its own gravity. Energy and matter are all consumed by the strength of the gravity creating the black hole.

### **Objectives:**

- Students will observe and demonstrate that the patterns of stars, for example, the constellations, stay the same in the night sky, although they appear to move across the sky with time.

- Students will determine that different star patterns can be seen in different seasons, and will use Greek myths to help locate and remember seasonal star arrangements in the sky.
- Students will organize and graph starlight data from the winter sky. Students will analyze and describe visible starlight in terms of star color, brightness, and temperature.
- Students will uncover evidence that the Sun is a medium-sized, main-sequence star as they create and interpret the Hertzsprung-Russell diagram, an essential tool of modern astronomers.

### **Skills:**

- Students will learn to use a planisphere of the night sky to discover nightly and seasonal star patterns.
- Students will be able to analyze patterns in the seasonal grouping of stars, and discover that story-telling was a convenient memory device used by many ancient cultures to locate important stars and star patterns in different seasonal skies.
- Students will know how to organize and manipulate starlight data to produce graphs (histograms). These histograms will assist their comparison of our own Sun to other stars.
- Students will learn to create and interpret the Hertzsprung-Russell diagram.

### **Vocabulary:**

- planisphere
- constellations
- Histograms
- color
- luminosity
- temperature (Kelvins)
- spectral class
- absolute magnitude
- red giants
- white dwarfs
- main sequence stars
- supernova
- neutron stars
- pulsars
- black holes
- circumpolar stars/constellations
- Andromeda Group
- seasonal stars/constellations
- Hertzsprung-Russell diagram

## Frameworks Connections

### Science and Technology:

#### Strand 1: Inquiry

**Standard:** p. 28

- Note/describe relevant details, patterns and relationships. Describe trends in data even when patterns are not exact.

- Represent data and findings using tables, models, demonstrations and graphs.

#### Strand 2: Domains of Science

**Standard:** Physical science, p. 47-48.

-Investigate and describe an understanding of visible electromagnetic radiation, which we generally call light, with reference to qualities such as color and brightness.

**Standard:** Earth and space science, p. 77-78

-Observe and demonstrate that the patterns of stars in the sky stay the same, although they appear to move across the sky nightly, and different stars can be seen in different seasons.

-Demonstrate evidence that the Sun is a medium-sized star located near the edge of a disk-shaped galaxy of stars, part of which can be seen as a growing band of light that spans the sky on a very clear night.

## Unit 4 Lesson Plans



### Lesson 1: Star Patterns

Nathaniel Bowditch loved his children and entertained their visits to his study easily, often rewarding them for good behavior with small constellations drawn in pen on their hands. (Susan Bowditch, 1997)

#### Objectives

- Students will know how to observe and demonstrate that patterns of stars, for example, the constellations, stay the same in the night sky, although they appear to move across the sky with time.

#### Skills:

- Students will learn how to use a planisphere of the night sky to discover nightly and seasonal star patterns.

#### Vocabulary:

- circumpolar stars/constellations
- planisphere
- constellation
- Polaris
- Cepheus
- Ursa Minor
- Ursa Major
- Cassiopeia
- Orion
- seasonal stars/constellations
- zodiac stars/constellations

#### Materials:

- planisphere  
(see attached sheet for instructions)
- student worksheet
- pencils
- tracing paper
- notebook
- tape

#### Procedure:

1. **Planispheres:** Create and distribute classroom set of planispheres (20-30). Each student should have one for his/her own use during class. These can be laminated for durability.

## 2. Class Challenge:

Whole class activity - Use the board or an overhead projector for responses:

- What is a planisphere (Star Finder)?
- Describe as many features as you can find on the planisphere.
- Why are the **cardinal directions** on the planisphere?
- Lift and then turn the wheel carefully inside of the black envelope. Describe your observations, include changes in daily and seasonal time, movement of the patterns inside of the circle, etc.
- What are these dots and lines called that spin in the circle?
- Find today's date. Place the time at 9PM, and describe what you will expect in the sky.

## 3. Individual Activity

- Place the planisphere on your desk with the north end facing away from you. Place a small piece of tape on the back to keep the envelope from slipping.
- Spin the white constellation wheel to match today's date at 9 PM.
- Trace the movement of several of the constellations as they complete a yearly cycle. The tracing paper will need a border to match its correct placement each time you remove it to turn the wheel two months forward.
- With the tracing paper covering your planisphere, trace the edges of the right-angled corner of the black planisphere (the title, Pacific Science Center, is in this corner). You now have a guide to guarantee the correct placement of the tracing paper each time you turn the wheel.
- Now, let's follow a few constellations from today's date, through the rest of the year. We will trace the constellation's position every two months to complete an annual cycle. Begin with **Cassiopeia**.
- You will have to remove your tracing paper each time you move the wheel, so be careful to match the tracing of the black corner to the actual corner.
- What did you discover from your tracing?
- Now let's try **Cepheus** and **Ursa Major (Big Dipper)**. What did your tracing produce?



- What constellation seems to be always in the center of these tracings? Why?
- Now trace **Ursa Minor (Little Dipper)**, being careful to tightly replace the wheel each time to get an accurate tracing. What star is at the tip of the handle? Does this star also move throughout the year, or does it appear to stay? Why? What is the name of this star?

#### 4. New Class Challenge:

- Do all of the constellations follow this annual pattern?
- Select several constellations to test your hypothesis. Try some within the double lines, example **Virgo, Leo, Cancer**, etc. Also try constellations on the edge, example **Hydra, Monoceros, Orion**, etc.

#### 5. Analysis

- What different star patterns did your tracings produce?
- Did any of these constellations change throughout the year? What actually changed? Why does this happen?
- If you were to group these constellations, what groups would you make? Why?
- Which constellations (and their stars) would you call Circumpolar constellations? [what does circumpolar mean?] Seasonal constellations? Zodiac constellations? [what does zodiac mean?]
- Summarize your observations of seasonal star/constellation movements.
- Do the same constellations create a pattern each night?
- In the next lesson, we will examine constellation movements during different seasons to compare/contrast the star patterns and relate them to popular Greek myths.

#### Handouts:

- Planisphere
- Student Sheets: Lesson 1. Star Patterns





# STAR FINDER HOLDER

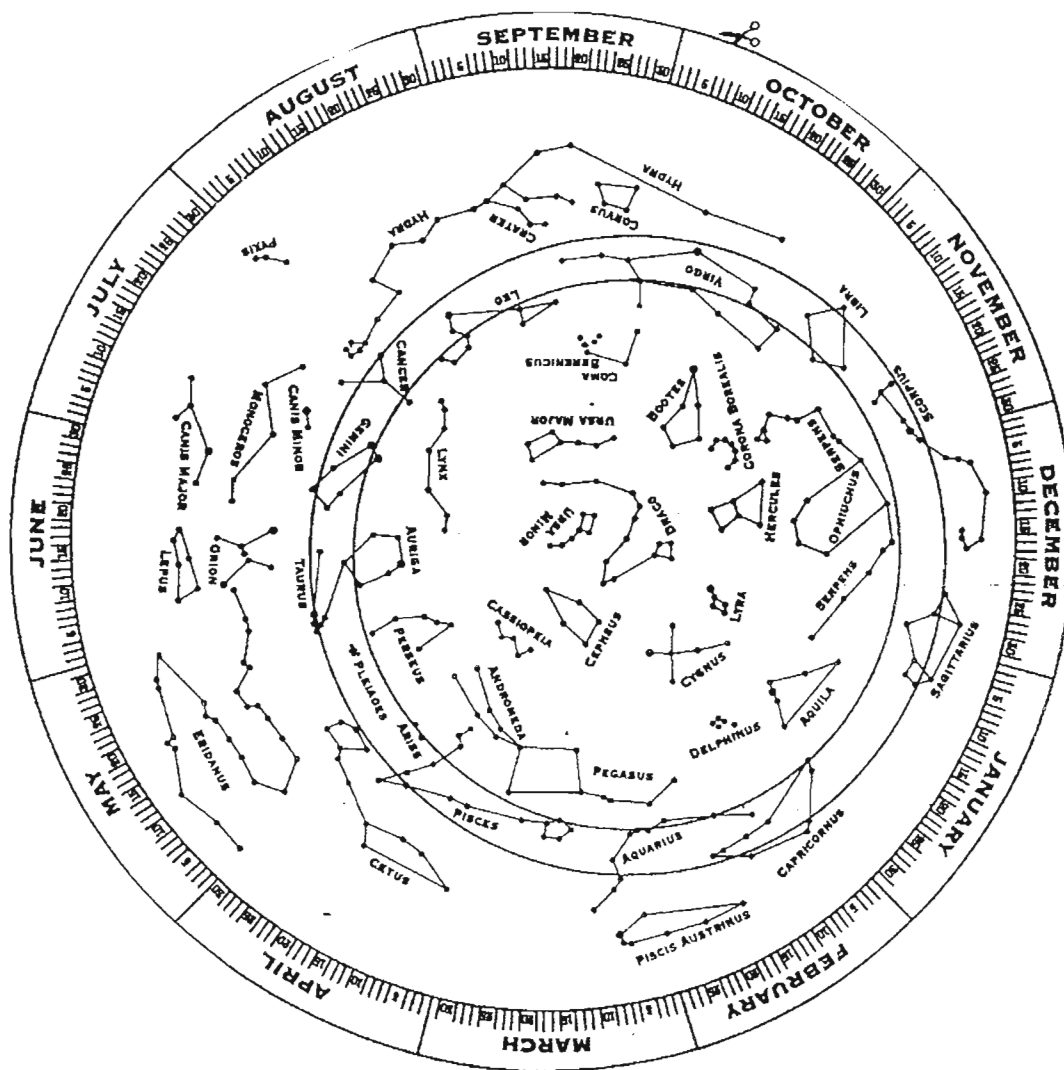


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# STAR WHEEL

## COMPLEX STAR FIELD



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# STAR PATTERNS

Answer the following questions as you work with the planisphere:

1. What is a planisphere?
2. Why are the cardinal directions on the planisphere?
3. Describe your observations as you turn the wheel - include daily changes, seasonal changes, movement of the constellations inside the view area, etc .
4. Find today's date. Place the time at 9 PM. Describe what you will see in tonight's sky.



## TRACINGS:

1. What did you discover from your first tracings of Cassiopeia?
2. Describe your next tracings of Cepheus and Ursa Major.
3. What constellation always seems to be in the center of these tracings? Why?
4. Trace Ursa Minor. What star is at the tip of the handle?
4. Does this star also move throughout the year, or does it appear to stay in one place? Why?
5. Why do we call this star Polaris?



## MORE TRACINGS AND ANALYSIS:

1. Do all of the constellations follow the pattern of Polaris and Ursa Minor? Test your hypothesis by tracing several constellations within the double line: Virgo, Leo, Cancer, etc. Also try constellations such as Hydra, Monoceros, Orion, etc.
2. What different star patterns did your tracings produce?
3. Did these constellations disappear any time throughout the year? What actually changed? Why?
4. If you were to group these constellations, what groups would you make? Why?
5. Which constellations and their stars would you call circumpolar constellations? Why?
6. Which constellations would you call seasonal constellations? Why?
7. Which constellations would you call zodiac constellations? Why?
8. Summarize your observations of seasonal star/constellation movements.







## Lesson 2: Seasonal Constellations & The Creation of Myths

### Objectives:

- Students will analyze patterns in the seasonal grouping of stars, and discover that the art of storytelling created a powerful memory device utilized by many cultures to locate and memorize important stars and star patterns (constellations) in different seasonal skies.

### Skills:

- Students will understand how to analyze patterns of stars.
- Students will be able to develop memorization skills through the story narrative.

### Vocabulary:

- |                   |              |             |
|-------------------|--------------|-------------|
| • Andromeda Group | • Cassiopeia | • Cepheus   |
| • Poseidon        | • Ethiopia   | • Andromeda |
| • Perseus         | • Pegasus    | • Medusa    |
| • Cetus           | • Pleides    |             |

### Materials:

- Student worksheet
- planisphere
- notebook
- Seasonal Sky Groups - handout

### Procedure:

#### 1. Group Challenge

- Do the constellations remain in the same part of the sky during each season? In your groups of four students, assign each student a different season to investigate. Select four different dates, one each for the four seasons, and trace several different constellations as they move from 7PM to 5AM.
- Compare your tracings. Did any dates contain the same pattern of stars?
- Can you identify the circumpolar constellations? the seasonal constellations? the zodiac constellations?
- Compare/contrast the star patterns you discovered in four different seasons. Can you see any differences, and if so, what might cause these

differences in constellations?

## **2. Class Activity:**

- If the star patterns are changing from one season to the next, how did people who depended on the stars for navigation, remember these changing patterns in the night sky?

**Hint:** What is one of the most ancient methods of passing important information from one generation to the next?

## **The Storyteller**

Stories are easy to remember. A good story contains a problem, a conflict, heroes, villains, and a final resolution. The rhythm of the story helps us remember and even learn new information better. The Greeks were one of the greatest storytellers of the ancient world. Let's listen to this myth while you read along, and see if you can find some familiar names:

### **Andromeda and Perseus**

Andromeda was a beautiful princess of Ethiopia. Her mother, the equally beautiful, but very vain Queen Cassiopeia, loved to boast a little too much about her own stunning appearance. This terrible vanity so angered the god, Poseidon, Ruler of the Seas, that he ordered the sea monster, Cetus, to wreck havoc on all coastal towns and villagers. Sailors were never safe as Cetus upset their boats, caused great storms, and put their lives in grave danger. Andromeda's father, King Cepheus, was not at all like his vain, self-centered wife. He became very concerned about the safety of his kingdom and his people, so he sought advice. Advice can come at a terrible price, however. The only solution to the dreadful sea monster's destruction was to sacrifice something of value and beauty. Only the most valued and beautiful girl of his kingdom would be able to calm the raging Posidon, and convince him to remove the monster, Cetus. Cepheus's daughter, Princess Andromeda, was the only choice as the most suitable sacrificial offering. Cepheus was devastated, but had no choice but to allow the offering to be carried out. He led his beautiful daughter down to the rocky coast where he left her chained to an outcropping. Alone she awaited the monster's appearance.

As the great monster approached Perseus suddenly arrived, riding the winged horse, Pegasus. Heroic Perseus had just completed the grim task of killing Medusa, one of the Gorgon sisters. Medusa, like her sisters, was a horrible sight to behold. If anyone even half glanced at her face, they would instantly be turned to stone. Perseus not only slayed Medusa, but placed her hideous head in a bag to prove his deed. Now seeing the beautiful girl's

danger, Perseus quickly pulled the grizzly head out from the leather bag, waved it in front of Cetus, and the horrible sea monster was instantly turned into a large mass of rock. Princess Andromeda was quickly rescued from the dangerous rocks and returned to her grieving parents. King Cepheus immediately granted Perseus's request to marry his daughter. As for the vain Queen Cassiopeia, she never again boasted about her beauty.

### **3. Class Analysis:**

- Who were the characters? What roles did they play?
- Now, let's create an hypothesis:

The Greeks were great storytellers, and storytelling is known to be an excellent way to remember past events as well as retain new information. Could there be a group of constellations with names like these in the story? If so, then repeating the story while gazing upwards at the star-filled sky would help someone remember this particular pattern of stars.

- Using your planisphere, look for the story's characters in the constellations.
- Turn the wheel to allow all of these constellation to be showing through your viewing circle. What months are these best seen in?
- When do you think Greek shepherds would most likely hear this story repeated as they gazed at the heavens each night?
- Now, here are some more seasonal constellation groups (see handout). Can you find these on your planisphere?
- Summarize your observations on myths and star patterns, or constellations, in the sky.

### **Independent Projects:**

- Adopt a constellation - Pick any of the constellations you have noticed on your planisphere, and create a colorful poster which contains:
  - the myth behind this star grouping
  - a tracing of it's yearly sky path, using the planisphere
  - the name and characteristics of the brightest stars contained in the constellation
  - a picture of this constellation surrounded by its nearest neighbors (seasonal group)

- a colorful picture of the character represented by your constellation.
- your own myth using any seasonal group of stars.
- a myth about the stars that are present on your birthday. Native American star myths are a good model for ideas.

### **Handouts**

- Seasonal Constellations Worksheet
- Seasonal Sky Groups



# SEASONAL CONSTELLATIONS

## Group Challenge:

1. Do the constellations remain in the same part of the sky during each season? In your group of four students, assign each student a different season to investigate. Select four different dates, one each for the four seasons, and trace several different constellations as they move from 7PM to 5AM.
2. Compare your tracings. Did any dates contain the same pattern of stars?
3. For your assigned season:  
  
Can you identify the circumpolar constellations?  
  
  
Can you identify the seasonal constellations?  
  
  
Can you identify the zodiac constellations?
4. Compare/contrast the constellations you discovered in four different seasons.

5. What might cause these differences in constellations?

**Class Activity:**

1. If the star patterns are changing from one season to the next, how did people who depended on the stars for navigation, for planting times, etc. remember these changing patterns in the night sky?

(Hint: What is one of the most ancient methods of passing on important information from one generation to the next?)

2. The Storyteller - Stories are easy to remember. A good story contains a problem, a conflict, heroes, villains, and a final resolution. The rhythm of the story helps us remember and even learn new information better. The Greeks were one of the greatest storytellers of the ancient world. Let's listen to this myth while you read along, and see if you can find some familiar names:



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Andromeda was a beautiful princess of Ethiopia. Her mother, the equally beautiful, but very vain Queen Cassiopeia, loved to boast a little too much about her own stunning appearance. This terrible vanity so angered the god, Poseidon, Ruler of the Seas, that he ordered the sea monster, Cetus, to wreck havoc on all coastal towns and villagers. Sailors were never safe as Cetus upset their boats, caused great storms, and put their lives in grave danger. Andromeda's father, King Cepheus, was not at all like his vain, self-centered wife. He became very concerned about the safety of his kingdom and his people, so he sought advice. Advice can come at a terrible price, however. The only solution to the dreadful sea monster's destruction was to sacrifice something of value and beauty. Only the most valued and beautiful girl of his kingdom would be able to calm the raging Poseidon, and convince him to remove the monster, Cetus. Cepheus's daughter, Princess Andromeda, was the only choice as the most suitable sacrificial offering. Cepheus was devastated, but had no choice but to allow the offering to be carried out. He led his beautiful daughter down to the rocky coast where he left her chained to an outcropping. Alone she awaited the monster's appearance.

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### **Class Analysis:**

- Who were the characters? What roles did they play?

- Now, let's create a hypothesis:

The Greeks were great storytellers, and storytelling is known to be an excellent way to remember past events as well as retain new information. Could there be a group of constellations with the same names like those in the story? If so, then, a reasonable hypothesis would be that repeating the story while gazing upwards at the star-filled sky would help someone remember this particular pattern of stars.

- Using your planisphere, look for the story's characters in the constellations.

- Turn the wheel to allow all of these constellations to be showing through your viewing circle. What months are these best seen in?

- When do you think Greek shepherds would most likely hear this story repeated as they gazed at the heavens each night?

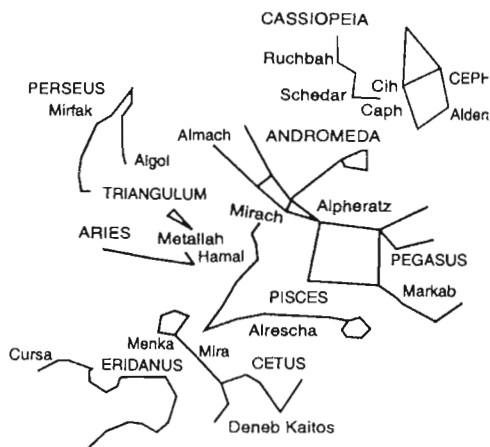
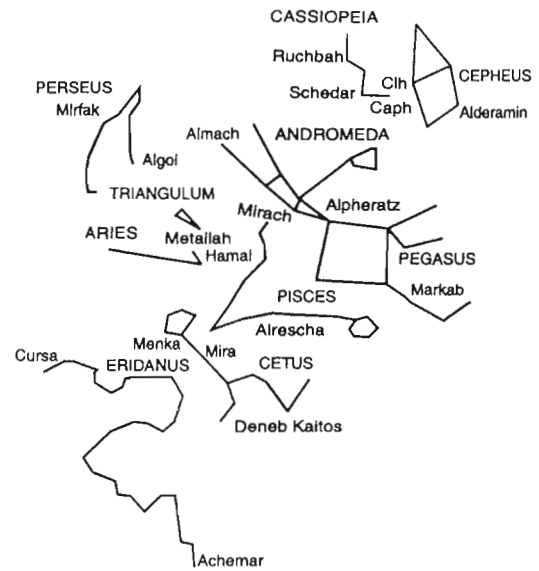
- Now, here are some more seasonal constellation groups (see handout). Can you find these on your planisphere?

- Summarize your observations and conclusions on myths and star patterns (constellations), in the sky.

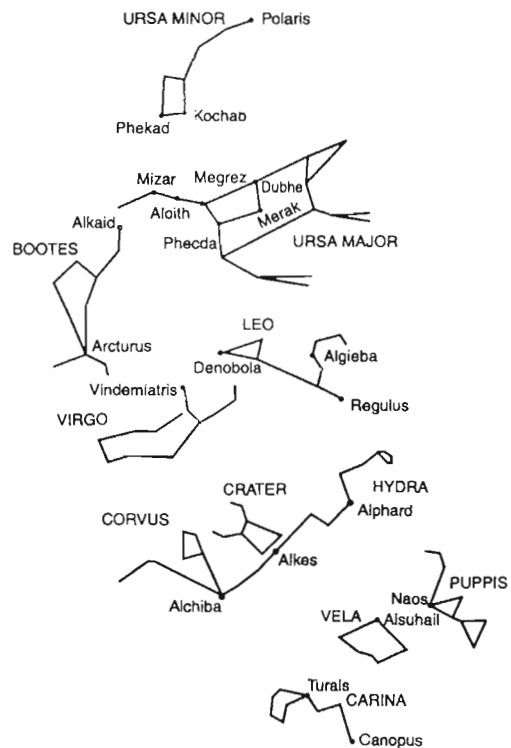




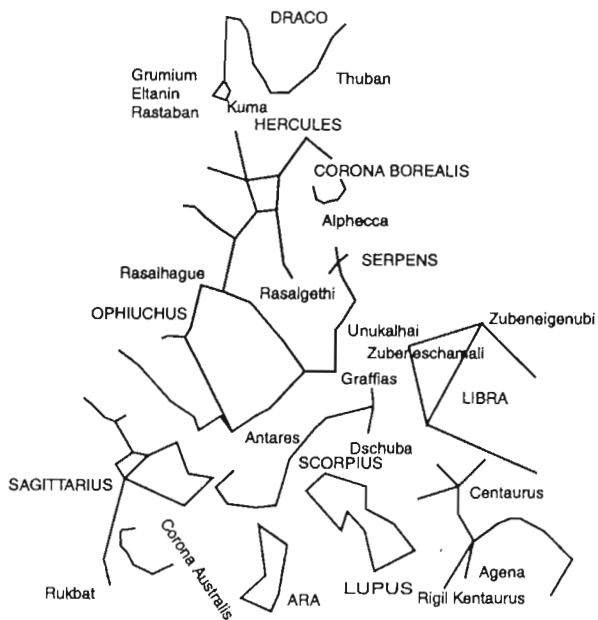
## The Andromeda Group



## The Osiris Group

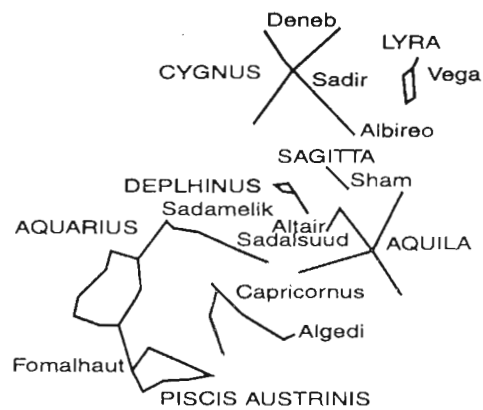


## The Harnessing & Harvesting Group



## The Hero and Healer Group

## The Birds and Sea Group





## Lesson 3: The Nature of Starlight

### Objectives:

- Students will compare/contrast our Sun's visible light characteristics with other stars by organizing and graphing star color, absolute magnitude, temperature, spectral type, luminosity, and diameter.

### Skills:

- Students will know how to organize and manipulate star data.
- Students will be able to produce and interpret histograms.
- Students will learn to compare and contrast quantitative data,

### Vocabulary:

- Histograms
- distance in light-years
- peak color in spectrum
- luminosity class
- diameter in Suns
- luminosity in Suns
- apparent magnitude
- temperature in Kelvins ( °K)
- absolute magnitude
- spectral type

### Materials:

- What is Starlight? - handout
- Questions for Starlight Jeopardy - handout
- Among the Stars of Winter Database - handout
- Definitions of Star Terms - handout
- Organizing the Star Data - handout
- Creating Histograms for Starlight Data - handout
- notebook
- centimeter graph paper
- metric ruler
- colored pencils

### Procedure:

#### Class Challenge (on board or overhead):

- Brainstorm: What is starlight? How do we describe it?
- After some discussion, reveal a list of all of the vocabulary words above, except for histograms.

- Challenge the students to try to define these words and then, explain how they relate to the question, What is starlight? Provide textbooks, dictionaries, etc. to encourage self-discovery.
- If needed for further discussion and clarification, distribute copies of the definitions provided in the handout.

### **Class Activity:**

- Play Starlight Jeopardy (10 minutes):
- Divide the class into groups of 4 to 6 students.
- Assign jobs - Recorder, Team Captains.
- Hand out the sheet, Among the Stars of Winter database.
- Read out loud a selected level of question, calling on the first person who raised his/her hand.
- If the answer is correct, assign the student's team the correct points. If incorrect, delete a point.
- Team captains must keep their group concentrating and working on each question as quickly as possible. If the groups become too noisy, they lose one point.
- The group with the highest score after ten minutes wins the game.

### **Next Class Challenge (on board or overhead):**

- How can we organize all of these numbers in the star database? (use board or overhead)
- Example: Look at the data column called Distance in Light-Years. How would you rearrange this long column to create a logical pattern?
- [after class brainstorming] You can organize it by resorting the data from the smallest number to the largest number.
- You can create groups of similar number.
- Hand out the student worksheet - Organizing the Star Data.

### **Group Activity:**

- Use the worksheets provided to begin organizing your new data.
- Do these special types of bar graphs, called histograms, help you understand your data better? How?

- What can you now conclude about the various stars in this data base and their distance in light-years from Earth?

#### **Group Activity:**

- Astronomers like to find patterns in their starlight data. For instance, the chart below suggests that stars have different colors, and the color of a star has something to do with how hot it is. It also suggests that the lifetime of a star is related to star temperature and color.

<b>Spectral Type</b>	<b>Temperature</b>	<b>Peak Color In</b>	<b>Lifetime</b>
O	Over 25,000	Blue	10 million years
B	11,000-25,000	Blue-White	40 million years
A	7,500-11,000	White	100 million years
F	6,000-7,500	Yellow-White	5 billion years
G (Sun)	5,000-6,000	Yellow	10 billion years
K	3,500-5,000	Orange	50 billion years
M	less than 3,500	Red	100 billion years

- Let's analyze this chart.
- Your new challenge is to create another set of histograms from your star data - temperature, peak color, and spectral type.
- Use the student worksheet, *Creating Histograms for Starlight Data*, to organize your data.
- In your group, graph these 3 histograms, assigning different graphs to each of your group members.

#### **Final Group Activity**

- Create your own histograms of absolute magnitude, luminosity, luminosity class, and diameter in suns.
- Answer the questions under Analysis.

**Handouts:**

- What is Starlight?
- Definitions of Starlight Variables
- Among the Stars of Winter Database
- The Meaning of Star Names
- Starlight Jeopardy Questions
- Organizing the Star Data
- Creating Histograms for Starlight Data.

## Starlight Jeopardy Answers

Each of the questions below is worth 1, 2 or 3 points, according to the difficulty of the question (1=easy, 2=moderate, 3=difficult). Ask each team to select the level of difficulty they would like to attempt. To keep it simple, continue on to the next group after one group has either attempted to answer the question, or answered it correctly. Students may use the Stars of Winter Database, the definition sheet and their notes. The correct answer is in parentheses. This short game is only a memory builder to introduce new vocabulary.

### 1 POINT QUESTIONS:

1. Distance in space is measured in \_\_\_\_\_ (light-years).
2. Starlight is studied by \_\_\_\_\_ (spectroscopy).
3. The surface temperature in stars is measured in the \_\_\_\_\_ (Kelvin) scale.
4. Star class is called the \_\_\_\_\_ (luminosity) class by astronomers.
5. The diameter of a star is the width of the star compared to the \_\_\_\_\_ (sun).
6. \_\_\_\_\_ (Luminosity) is the total light energy emitted by the star, as compared to the sun.
7. The true brightness of a star measures the stars as if they were all the same distance away (32.6 light-years). \_\_\_\_\_ (absolute magnitude).
8. O,B,A,F,G,K, and M are called \_\_\_\_\_ (spectral) classifications.

### 2 POINT QUESTIONS:

1. Describe the logarithmic magnitude scale.
2. The temperature of Sirius is 9,700 K. What is this in Celsius?
3. What is the name of the apparently brightest star in the database?
4. The star that is truly the brightest in our database is \_\_\_\_\_?
5. The star, Almaaz, has a diameter how many times greater than our Sun?
6. The hottest star in the database is \_\_\_\_\_?

7. Explain the spectral type classification for the star, Capella.
8. The furthest star from our Sun is \_\_\_\_\_.
9. The star with the greatest total light energy emitted is \_\_\_\_\_.
10. Our Sun's luminosity class, or star's class, is \_\_\_\_\_.

### **3 POINT QUESTIONS:**

1. Explain the classification of stars according spectral type.
2. Explain the scale measuring the absolute magnitude of stars.
3. Betelgeuse has a luminosity value of 5,000. What does this mean?
4. The star, Wezen, has a diameter value of 365, and Sirius has a diameter value of 2. What do these numbers mean?
5. The spectral type for our Sun is G2 V. What does this mean?
6. Describe the stages, or luminosity classes, of stars.
7. Describe the Kelvin scale for measuring temperature of stars.
8. How far (in kilometers) does light travel from Sirius to our Earth?
9. What star name means "camels quenching their thirst"?
10. Which star is often called the "dog star"?





# WHAT IS STARLIGHT?

1. Brainstorm ideas to the question, What Is Starlight?

2. Define these vocabulary words provided by your teacher. Use text books, dictionaries, Internet resources, etc. to discover the definitions.

- Distance in light-years
- Peak color in spectrum
- Luminosity class
- Temperature in Kelvins (K)
- Diameter in Suns
- Luminosity in Suns
- Apparent magnitude
- Absolute magnitude
- Spectral type

## Definitions of Starlight Variables

1. **Distance:** Distance in space is measured in light-years. One light-year is the distance light travels in a year, or about 9.5 trillion kilometers in one year, or about 6 trillion miles in one year.
2. **Peak Color in Spectrum:** Starlight is studied by spectroscopy (using diffraction to break light into its component colors). Depending on how hot a star is, the light emitted from the star shines brightest in certain wavelengths. Stars whose spectra peak in the red are cooler than stars whose spectra peak in the blue.
3. **Temperature (Kelvins):** The surface temperature of the star. When the stars are arranged by surface temperature and by peak color, the relationship between the two is easily seen.
4. **Star's Luminosity Class:** This represents the stage of the star's life cycle. For instance, most stars spend most of their existence in the main sequence phase. Later, stars enlarge dramatically to become giant or supergiant stars. Finally, most stars shrink to become white, red, or black dwarfs. Some stars explode as supernovae while their cores collapse into extremely dense neutron stars or even black holes.
5. **Diameter in Suns:** The width of the star, as compared to our Sun.
6. **Luminosity:** The total light energy emitted by the star, as compared to our Sun.
7. **Absolute Magnitude:** The true brightness of a star; this scale measures the stars as if they were all the same distance away (about 32.6 light years). The smaller numbers indicate brighter stars; zero and negative numbers indicate still greater brightness.
8. **Spectral Type:** Spectral classifications are O, B, A, F, G, K, and M. O stars are the hottest and M stars are the coolest. Luminosity class is indicated by Roman numerals. I is a supergiant; II is bright giant; III is giant; IV is subgiant; and V is main sequence. Spectral and luminosity classes are further subdivided with numbers and letters.



# THE STARS OF WINTER DATABASE

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Star Name	Navigation Star ☆	Distance in Light-Years	Peak Color in Spectrum	Star's Luminosity Class	Temperature in Kelvin (K)	Diameter in Suns	Luminosity in Suns	Absolute Magnitude	Spectral Type
Capella	☆☆	44	yellow	giant	5100	11	72	0.9	GB III
Menkalinn		80	blue-white	subgiant	9000	2	45	0.6	A2 IV
Almaaz		6500	white	supergiant	7200	365	200000	-8.5	F0 Ia
Hoedus II		310	blue	main sequence	21000	3	377	-1.7	B3 V
Hassaleh		330	orange	bright giant	4200	73	655	-2.3	K3 II
Theta Auriga		150	blue-white	peculiar	10000	2	146	-0.7	A0 pec
Hoedus I		530	orange	bright giant	4300	53	655	-2.3	K4 II
Sirius	☆☆	9	blue-white	main sequence	9700	2	21	1.42	A1 V
Mirzam		740	blue	bright giant	26000	4	6500	-4.8	B1 II
Wezen		3100	white	supergiant	6000	365	125000	-8	F8 Ia
Adhara	☆☆	490	blue	bright giant	20000	5	4500	-4.4	B2 II
Muliphen		1000	blue	bright giant	14000	5	1803	-3.4	B8 II
Aludra		2500	blue	supergiant	14500	37	50000	-7	B5 Ia
Furud		290	blue	main sequence	18000	2	377	-1.7	B2.5 V
Procyon	☆☆	11	white	subgiant	6700	2	7	2.64	F5 IV
Gomeisa		140	blue	main sequence	13000	2	95	-0.2	B8 Ve
Castor		47	blue-white	main sequence	9300	2	28	1.14	A1 V
Pollux	☆☆	35	orange	giant	4900	9	32	0.98	K0 IIIb
Wasat		53	white	subgiant	7000	2	8	2.46	F2 IV
Mebuta		190	yellow	supergiant	5000	33	175	-0.9	G8 Ib
Alhena		88	blue-white	subgiant	9800	3	79	0	A0 IV
Propus		190	red	giant	3100	34	125	-0.5	M3 III
Tejat Posterior		160	red	giant	2900	35	125	-0.5	M3 IIIa
Alzirr		59	white	giant	6600	2	11	2.1	F5 III
Mekbuda		1500	yellow	supergiant	5700	86	5000	-4.5	G0 Ib
Sun	☆☆	8.3 light-minutes	yellow	main sequence	5800	1	1	4.75	G2 V



# THE STARS OF WINTER DATABASE 2

Star Name	Navigation Star ☆	Distance in Light-Years	Peak Color in Spectrum	Star's Luminosity Class	Temperature in Kelvins (K)	Diameter in Suns	Luminosity in Suns	Apparent Magnitude	Spectral Type
Ameb		930	white	supergiant	7400	32	6000	-4.7	F0 Ib
Nihal		320	yellow	bright giant	5600	30	545	-2.1	G5 II
Betelgeuse	☆☆	325	red	supergiant	3400	265	5000	-4.5	M1 lab
Rigel	☆☆	910	blue	supergiant	13000	58	55000	-7.1	B8 lac
Mintaka		2300	blue	giant	24000	13	50000	-7	BO III
Ainilam	☆☆	1200	blue	supergiant	23000	16	25000	-6.2	BO lae
Bellatrix	☆☆	360	blue	giant	23000	3	2168	-3.6	B2 III
Algibba		770	blue	main sequence	19000	8	1977	-3.5	B1 V
Nair Al Saif		1900	blue	giant	28000	6	20000	-6	O9 III
Saiph		215	blue	supergiant	22000	4	525	-2.1	BO.5 Ia
Meissa		470	blue	not identified	35000	3	552	-2.2	O8 e
Alnitak		1600	blue	supergiant	28000	80	34000	-6.6	O9.5 Ib
Aldebaran	☆☆	65	orange	giant	4000	34	137	-0.6	K5 III
El Nath	☆☆	150	blue	giant	14000	2	344	-1.6	B7 III
Ainilam		150	yellow	giant	5000	13	65	0.2	G9.5 III
Al Hecka		520	blue	giant	18000	4	1247	-3	B4 III
Alcyone		260	blue	giant	15000	3	344	-1.6	B7 III
Sun	☆☆	8.3 light- minutes	yellow	main sequence	5800	1	1	4.75	G2 V

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# THE MEANINGS OF STAR NAMES

Constellation Name	Star Name	Meaning of Star Name
Auriga, the Chariteer	Capella	little she-goat, goat star, rainy goat star
Auriga, the Chariteer	Menkaliman	shoulder of the rein holder
Auriga, the Chariteer	Almaaz	he-goat; western goat str; signal for close of navigation; also called Al Anz
Auriga, the Chariteer	Hoedus II	one of kid goats, rising before Sun, marks stormy season
Auriga, the Chariteer	Hasseleh	marks back of charioteer's knee
Auriga, the Chariteer	Theta Auriga	marks wrist of charioteer
Auriga, the Chariteer	Hoedus I	one of kid goats; rising before Sun, marks stormy season; also called Sadatoni
Canis Major, the Great Dog	Sirius	sparkling; dog star; scorching one; rising before Sun on hottest days of summer
Canis Major, the Great Dog	Mirzam	roarer or announcer (of Sirius)
Canis Major, the Great Dog	Wezen	weight; also called Wesen
Canis Major, the Great Dog	Adhara	maiden, attendant of Suhail who married Orion
Canis Major, the Great Dog	Muliphen	marks the top of the dog's head
Canis Major, the Great Dog	Aludra	maiden, attendant of Suhail who married Orion
Canis Major, the Great Dog	Furud	male apes, also called Phurud
Canis Minor, Small Dog	Procyon	before the dog (rising before Sirius), water dog (near Milky Way)
Canis Minor, Small Dog	Gomeisa	watery eyed (near Milky Way), also called Mirzam
Gemini, The Twins	Castor	horseman, mortal twin
Gemini, The Twins	Pollux	boxer, immortal twin
Gemini, The Twins	Wasat	middle of the sky (near the ecliptic)
Gemini, The Twins	Mebstata	outstretched paw of the lion
Gemini, The Twins	Alhena	brand mark
Gemini, The Twins	Propus	the projecting foot; also called Tejat Prior
Gemini, The Twins	Tejat Posterior	heel
Gemini, The Twins	Alzirr	button
Gemini, The Twins	Mekbuda	folded paw of the lion



## THE MEANINGS OF STAR NAMES PAGE 2

Lepus, The Hare	Arneb	the hare
Lepus, The Hare	Nihal	camels quenching their thirst
Orion, The Hunter	Betelgeuse	arm of central one; armpit of white belted sheep
Orion, The Hunter	Rigel	left leg of giant, Orion's left foot
Orion, The Hunter	Mintaka	belt
Orion, The Hunter	Ainilam	string of pearls
Orion, The Hunter	Bellatrix	Amazon female warrior
Orion, The Hunter	Algibba	handle of the sword
Orion, The Hunter	Nair al Saif	bright one of the sword
Orion, The Hunter	Saiph	sword of powerful one
Orion, The Hunter	Meissa	glittering star
Orion, The Hunter	Alnitak	girdle
Taurus, The Bull	Aldebaran	follower (of the Pleiades)
Taurus, The Bull	El Nath	the one butting with horns
Taurus, The Bull	Ain	eye
Taurus, The Bull	Al Hecka	white one
Taurus, The Bull	Alcyone	brightest one of the Pleiades (Seven Sisters)

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# STARLIGHT JEOPARDY

Each of the questions below is worth 1, 2 or 3 points, according to the difficulty of the question (1=easy, 2=moderate, 3=difficult). Ask each team to select the level of difficulty they would like to attempt. To keep it simple, continue on to the next group after one group has either attempted to answer the question, or answered it correctly. Students may use the Stars of Winter Database, the definition sheet and their notes. The correct answer is in parentheses. This short game is only a memory builder to introduce new vocabulary.

## 1 POINT QUESTIONS:

1. Distance in space is measured in \_\_\_\_\_ .
2. Starlight is studied by \_\_\_\_\_ .
3. The surface temperature in stars is measured in the \_\_\_\_\_ scale.
4. Star class is called the \_\_\_\_\_ class by astronomers.
5. The diameter of a star is the width of the star compared to the \_\_\_\_\_ .
6. \_\_\_\_\_ is the total light energy emitted by the star, as compared to the sun.
7. The true brightness of a star measures the stars as if they were all the same distance away (32.6 light-years). \_\_\_\_\_ .
8. O,B,A,F,G,K, and M are called \_\_\_\_\_ classifications.

## 2 POINT QUESTIONS:

1. Describe the logarithmic magnitude scale.
2. The temperature of Sirius is 9, 700 K. What is this in Celsius?
3. What is the name of the apparently brightest star in the database?
4. The star that is truly the brightest in our database is \_\_\_\_\_?
5. The star, Almaaz, has a diameter how many times greater than our Sun?
6. The hottest star in the database is \_\_\_\_\_?

7. Explain the spectral type classification for the star, Capella.
8. The furthest star from our Sun is \_\_\_\_\_.
9. The star with the greatest total light energy emitted is \_\_\_\_\_.
10. Our Sun's luminosity class, or star's class, is \_\_\_\_\_.

### **3 POINT QUESTIONS:**

1. Explain the classification of stars according spectral type.
2. Explain the scale measuring the absolute magnitude of stars.
3. Betelgeuse has a luminosity value of 5,000. What does this mean?
4. The star, Wezen, has a diameter value of 365, and Sirius has a diameter value of 2. What do these numbers mean?
5. The spectral type for our Sun is G2 V. What does this mean?
6. Describe the stages, or luminosity classes, of stars.
7. Describe the Kelvin scale for measuring temperature of stars.
8. How far (in kilometers) does light travel from Sirius to our Earth?
9. What star name means "camels quenching their thirst"?
10. Which star is often called the "dog star"?





## ORGANIZING THE STAR DATA

How can we organize and make sense of all of the numbers in the Stars of Winter Database?

Begin with one variable. For example, *Distance in light-years*.

- a. Rearrange your data by sorting it from the smallest number to the largest number.

Star Name	Distance in Light Years

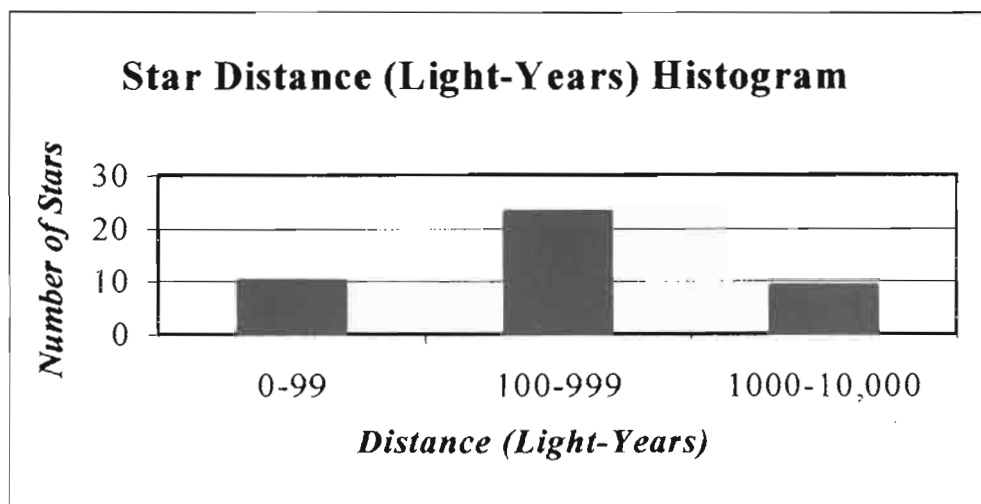
Star Name	Distance in Light Years

2. Next, create groups of similar numbers:

For example, the *Distance in light-years* data is easily grouped into three sections: 0 to 99, 100 to 999, and 1,000 to 10,000. Tally the number of stars in each group below:

0 to 99 light years	100 to 999 light-years	1000 to 100,000 light years

Your final bar graph, or histogram, is now very simple. Place the number of stars tally on the vertical axis and the three groups of stars on the horizontal axis:



3. What can you conclude about star distance from your histogram?



# CREATING HISTOGRAMS FOR STARLIGHT DATA

## Introduction

Astronomers like to find patterns in starlight data. The chart below suggests that stars can be organized by spectral type, temperature, and color. The column, lifetime, appears to be related to the other three variables as well.

Spectral	Type Temperature ( °K)	Peak Color in Spectrum	Lifetime
O	Over 25,000	Blue	10 million years
B	11,000-25,000	Blue-white	40 million years
A	7,500-11,000	White	100 million years
F	6,000-7,500	Yellow-white	5 billion years
G (Sun)	5,000-6000	Yellow	10 billion years
K	3,500-5000	Orange	50 billion years
M	less than 3,500	Red	100 billion years

1. Let's use these relationships to analyze our Stars of Winter database. We can create histograms to place our stars into like groups. First, sort the data in ascending order (smallest to largest). This has already been done for you below for temperature:

Star Name	Temperature in Kelvins (K)	Spectral Type	Peak Color in Spectrum
Tejat Posterior	2900	M3 IIIa	red
Propus	3100	M3 III	red
Betelgeuse	3400	M1 lab	red
Aldebaran	4000	K5 III	orange
Hassaleh	4200	K3 II	orange
Hoedus I	4300	K4 II	orange
Pollux	4900	KO IIIb	orange
Mebsuta	5000	G8 Ib	yellow
Ainilam	5000	G9.5 III	yellow
Capella	5100	GB III	yellow
Nihal	5600	G5 II	yellow
Mekbuda	5700	G0 Ib	yellow
Sun	5800	G2 V	yellow

Star Name	Temperature in Kelvins (K)	Spectral Type	Peak Color in Spectrum
Wezen	6000	F8 Ia	yellow-white
Alzirr	6600	F5 III	yellow-white
Procyon	6700	F5 IV	yellow-white
Wasat	7000	F2 IV	yellow-white
Almaaz	7200	F0 Ia	yellow-white
Ameb	7400	FO 1b	yellow-white
Menkalinin	9000	A2 IV	white
Castor	9300	A1 V	white
Sirius	9700	A1 V	white
Alhena	9800	A0 IV	white
Theta Auriga	10000	A0 pec	white
Gomeisa	13000	B8 Ve	blue-white
Rigel	13000	B8 lac	blue-white
Muliphen	14000	B8 II	blue-white
El Nath	14000	B7 III	blue-white
Aludra	14500	B5 Ia	blue-white
Alcyone	15000	B7 III	blue-white
Furud	18000	B2.5 V	blue-white
Al Hecka	18000	B4 III	blue-white
Algiebba	19000	B1 V	blue-white
Adhara	20000	B2 II	blue-white
Hoedus II	21000	B3 V	blue-white
Saiph	22000	BO.5 Ia	blue-white
Ainilam	23000	BO lae	blue-white
Bellatrix	23000	B2 III	blue-white
Mintaka	24000	BO III	blue-white
Mirzam	26000	B1 II	blue
Nair Al Saif	28000	O9 III	blue
Alnitak	28000	O9.5 Ib	blue
Meissa	35000	O8 e	blue

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2. Now, organize your large data set into groups of similar numbers.

- For example, using the chart at the beginning of this worksheet, spectral type groups (O,B,A,F,G,K,M), temperature groups (over 25, 000; 11,000-25,000 etc), and peak color in spectrum groups (blue, blue-white, etc) provide logical arrangements of the data.

*Simple tally the number of stars in each group:*

### Spectral Type

O	B	A	F	G	K	M

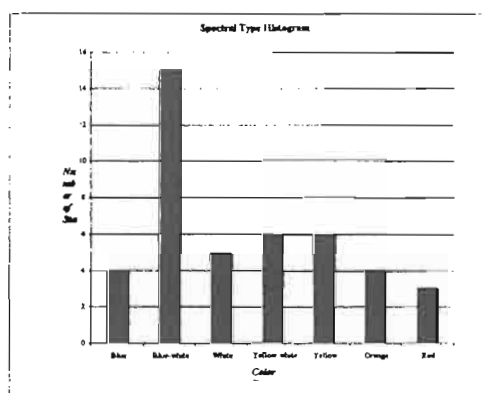
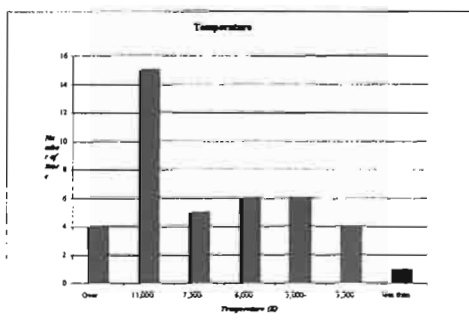
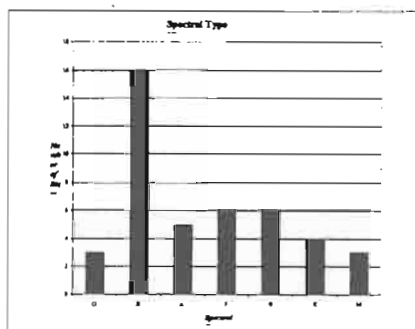
### Temperature (K)

Over 25,000	11,000 -25,000	7,500 -11,000	6,000 -7,500	5,000 -6,000	3,500 -5,000	Less than 3,500

### Peak Color in Spectrum

Blue	Blue -white	White	Yellow -white	Yellow	Orange	Red

Create 3 histograms, one for each variable above. Organize your graph with the vertical axis defined as the number of stars (your tally) in a group, and the horizontal axis as the group definition. Example:



- Now you need to create your final set of histograms for absolute magnitude, luminosity (in suns), star luminosity class, and diameter (in suns). The data is provided below, sorted from the brightest star to the dimmest star in the Stars of Winter Database.

Star Name	Absolute Magnitude	Luminosity in Suns	Star's Luminosity Class	Diameter in Suns
Almaaz	-8.5	200000	supergiant	365
Wezen	-8	125000	supergiant	365
Rigel	-7.1	55000	supergiant	58
Aludra	-7	50000	supergiant	37
Mintaka	-7	50000	giant	13
Alnitak	-6.6	34000	supergiant	80
Ainilam	-6.2	25000	supergiant	16
Nair Al Saif	-6	20000	giant	6
Mirzam	-4.8	6500	bright giant	4
Ameb	-4.7	6000	supergiant	32
Mekbuda	-4.5	5000	supergiant	86
Betelgeuse	-4.5	5000	supergiant	265
Adhara	-4.4	4500	bright giant	5
Bellatrix	-3.6	2168	giant	3
Algiebba	-3.5	1977	main sequence	8
Muliphen	-3.4	1803	bright giant	5
Al Hecka	-3	1247	giant	4
Hassaleh	-2.3	655	bright giant	73
Hoedus I	-2.3	655	bright giant	53
Meissa	-2.2	552	not identified	3
Nihal	-2.1	545	bright giant	30
Saiph	-2.1	525	supergiant	4
Hoedus II	-1.7	377	main sequence	3
Furud	-1.7	377	main sequence	2
El Nath	-1.6	344	giant	2
Alcyone	-1.6	344	giant	3
Mebsuta	-0.9	175	supergiant	33
Theta Auriga	-0.7	146	peculiar	2
Aldebaran	-0.6	137	giant	34
Propus	-0.5	125	giant	3
Tejat Posterior	-0.5	125	giant	35
Gomeisa	-0.2	95	main sequence	2
Alhena	0	79	subgiant	3
Ainilam	0.2	65	giant	13
Menkalinin	0.6	45	subgiant	2
Capella	0.9	72	giant	11
Pollux	0.98	32	giant	9
Castor	1.14	28	main sequence	2
Sirius	1.42	21	main sequence	2
Alzirr	2.1	11	giant	2
Wasat	2.46	8	subgiant	2
Procyon	2.64	7	subgiant	2
Sun	4.75	1	main sequence	1

4. Again, the data must be grouped in some meaningful way. A scale that increases each time by a factor of ten is useful when dealing with very large numbers. This is called a log scale.

5. We will use a log scale to group luminosity, and repeat these divisions for absolute value (also a measure of brightness). Luminosity class will be grouped by class name. Diameter (in suns) will also use a log scale to distinguish groups of stars.

6. The different groupings are provided in tables below to help you create your histograms:

### Luminosity (in suns)

Over 100,000	10,000 -100,000	1,000 -10,000	100 -1,000	10 -100	0 -10

### Absolute Magnitude

Less than -7.9	-7.9 to -5.9	-5.9 to -2.9	-2.9 to -0.49	-0.49 to 2.1	2.1 to 4.65

### Star Luminosity Class

Super Giant	Giant	Bright Giant	Sub Giant	Main Sequence	Peculiar	Not Identified

### Diameter (in suns)

1-10	10-100	100-1000



7. Using graph paper, create your histograms for the four variables above.  
Use the histograms for peak color, temperature, and spectral type as models

**Analysis Questions:**

- How is a histogram different from a line graph?
- When analyzing large databases, explain why histograms are a useful first step in identifying meaningful patterns.
- Compare and contrast your results from graphing star temperature, peak color in the spectrum, and spectral type.
- Compare and contrast our Sun's temperature, color, and spectral type to the other stars in the database.

- Compare and contrast our Sun's luminosity and absolute magnitude to the other stars in the database.
- What is our Sun's luminosity class?
- Write a paragraph describing our Sun according to the results of your analysis of the Stars of Winter Database. What kind of star is our Sun?



## Lesson 4: The Hertzsprung-Russell Diagram (H-R Diagram)

### Objectives:

- Students will uncover further evidence that the Sun is a medium-sized, main-sequence star as they create and interpret the Hertzsprung-Russell diagram, an essential tool of modern astronomers.

### Skills:

- Students will understand how to graph star data
- Students will be able to organize numerical data into meaningful groups
- Students will learn to analyze the significance of their results

### Vocabulary:

- Hertzsprung-Russell diagram
- Main sequence stars
- Red Giants
- White Dwarfs
- Neutron Stars
- Supernova
- Black Holes
- Star Life Cycle or Evolution

### Materials:

- Hertzsprung-Russell Diagram - handout
- Star Data for Creating the H-R Diagram
- centimeter graph paper
- metric ruler
- notebook
- colored pencils, markers, etc.

### Procedure:

#### Class Challenge:

You are now ready to create the single most important diagram astronomers use today, the Hertzsprung-Russell (H-R) diagram.

After you create your diagram, we will spend time analyzing the important star patterns it contains, and research how this diagram explains the natural life cycle, or birth, death, and re-birth of stars.

**Individual Activity:**

- You will be creating your own graph using the Star Data for Creating the H-R Diagram.
- On graph paper, create a horizontal scale (x-axis) for Temperature (K). Your data will begin with the hottest stars at the origin, and decrease in temperature from left to right. Your data will range from 23,000 K to 2,000 K.
- Create your vertical scale (y-axis) for Absolute Magnitude. Your data will begin with the most positive numbers at the origin (+15.0) to the most negative numbers at the top of the vertical axis (-8.0).
- Plot each star point for absolute magnitude and temperature. Do not connect the data points.
- Write the name of each star next to its dot on the diagram.
- Find the chart in this handout that contains information on spectral type, temperature, and color. Lightly shade in the color of each spectral type where it corresponds to a range of temperatures.

**Class Activity: The Life Cycle or Evolution of Stars**

But...how do we create a meaningful pattern from this scattering of data?

We go back to the drawing board!

- The creation of a scatter plot of different stars according to their absolute magnitude and temperature, as well as an overlay of star color, was a major discovery in modern astronomy. The interpretation was improved even further when the star's luminosity class was identified. This natural grouping of the data led two scientists in the early 1900s, Danish astronomer Ejnar Hertzsprung and American astronomer Henry Norris Russell to create what is now known as the Hertzsprung-Russell (H-R) diagram.
- What is the relationship between star color and star temperature? Between star absolute magnitude and luminosity?

- Now for a little group research:

In your groups, define these terms and explain how they relate to a star's life cycle:

- a main sequence star?
- a red giant?
- a white dwarf?
- a supernova?
- a neutron star?
- a black hole?

- What kind of star is our Sun?

- Use the chart below to determine the oldest stars and the youngest stars in your Hertzsprung-Russell diagram:

Spectral	Type Temperature ( °K)	Peak Color in Spectrum	Lifetime
O	Over 25,000	Blue	10 million years
B	11,000-25,000	Blue-white	40 million years
A	7,500-11,000	White	100 million years
F	6,000-7,500	Yellow-white	5 billion years
G (Sun)	5,000-6000	Yellow	10 billion years
K	3,500-5000	Orange	50 billion years
M	less than 3,500	Red	100 billion years

Oldest Stars

Youngest Stars

**Analysis:**

- Using either your H-R diagram, or a more complete H-R diagram you may have found during your research, summarize the natural evolution of stars from birth to death to rebirth. Include your understanding of the changes in absolute magnitude, temperature, and color as a star progresses through this cycle.

**Handouts**

- Creating the Hertzsprung-Russell Diagram
- Data for the Hertzsprung-Russell Diagram



# CREATING THE HERTZSPRUNG-RUSSELL DIAGRAM

## Class Challenge:

1. You are now ready to create the single most important diagram astronomers use today, the Hertzsprung-Russell (H-R) diagram.

After you create your diagram, we will spend time analyzing the important star patterns it contains, and research how this diagram explains the natural life cycle, or birth, death, and re-birth, of stars.

## Individual Activity:

2. You will be creating your own graph using the Star Data for Creating the H-R Diagram.
3. On graph paper, create a horizontal scale (x-axis) for Temperature (K). Your data will begin with the hottest stars at the origin, and decrease in temperature from left to right. Your data will range from 23,000 K to 2,000 K.
4. Create your vertical scale (y-axis) for Absolute Magnitude. Your data will begin with the most positive numbers at the origin (+15.0) to the most negative numbers at the top of the vertical axis (-8.0).
5. Plot each star point for absolute magnitude and temperature. Do not connect the data points.
6. Write the name of each star next to its dot on the diagram.
7. Find the chart in this handout that contains information on spectral type, temperature, and color. Lightly shade in the color of each spectral type where it corresponds to a range of temperatures.



## CLASS ACTIVITY: THE LIFE CYCLE OF STARS

1. But...how do we create a meaningful pattern from this scattering of data?

We go back to the drawing board!

2. The creation of a scatter plot of different stars according to their absolute magnitude and temperature, as well as an overlay of star color, was a major discovery in modern astronomy. This discovery was further improved when the star's luminosity class was included. This natural grouping of the data led two scientists in the early 1900s, Danish astronomer Ejnar Hertzsprung and American astronomer Henry Norris Russell to create what is now known as the Hertzsprung-Russell (H-R) diagram.
3. What does the H-R diagram suggest about the relationship between absolute magnitude and star temperature?
4. What does the H-R diagram suggest about the relationship between star spectral type and temperature?



5. Now for a little group research:

In your groups,

- Research and define the following terms.
  - Explain how they are related to a star's life cycle.
- a main sequence star?

a red giant?

a white dwarf?

a supernova?

a neutron star?

a black hole?

6. Can you identify any of these star types in your graph? For instance, where would you expect to find the main sequence stars?

-the red giants and supergiants?

-the white dwarves?

7. What kind of star is our Sun?

8. Use the chart below to determine the oldest stars and the youngest stars in your Hertzsprung-Russell diagram:

Spectral	Type Temperature ( °K)	Peak Color in Spectrum	Lifetime
O	Over 25,000	Blue	10 million years
B	11,000-25,000	Blue-white	40 million years
A	7,500-11,000	White	100 million years
F	6,000-7,500	Yellow-white	5 billion years
G (Sun)	5,000-6000	Yellow	10 billion years
K	3,500-5000	Orange	50 billion years
M	less than 3,500	Red	100 billion years

Oldest Stars

Youngest Stars

### Analysis:

Use either your H-R diagram, or even better, a more complete H-R diagram you may have found during your research, to summarize the natural evolution of stars from birth to death to rebirth. Include your understanding of the changes in absolute magnitude, temperature, and color as a star progresses through this cycle.

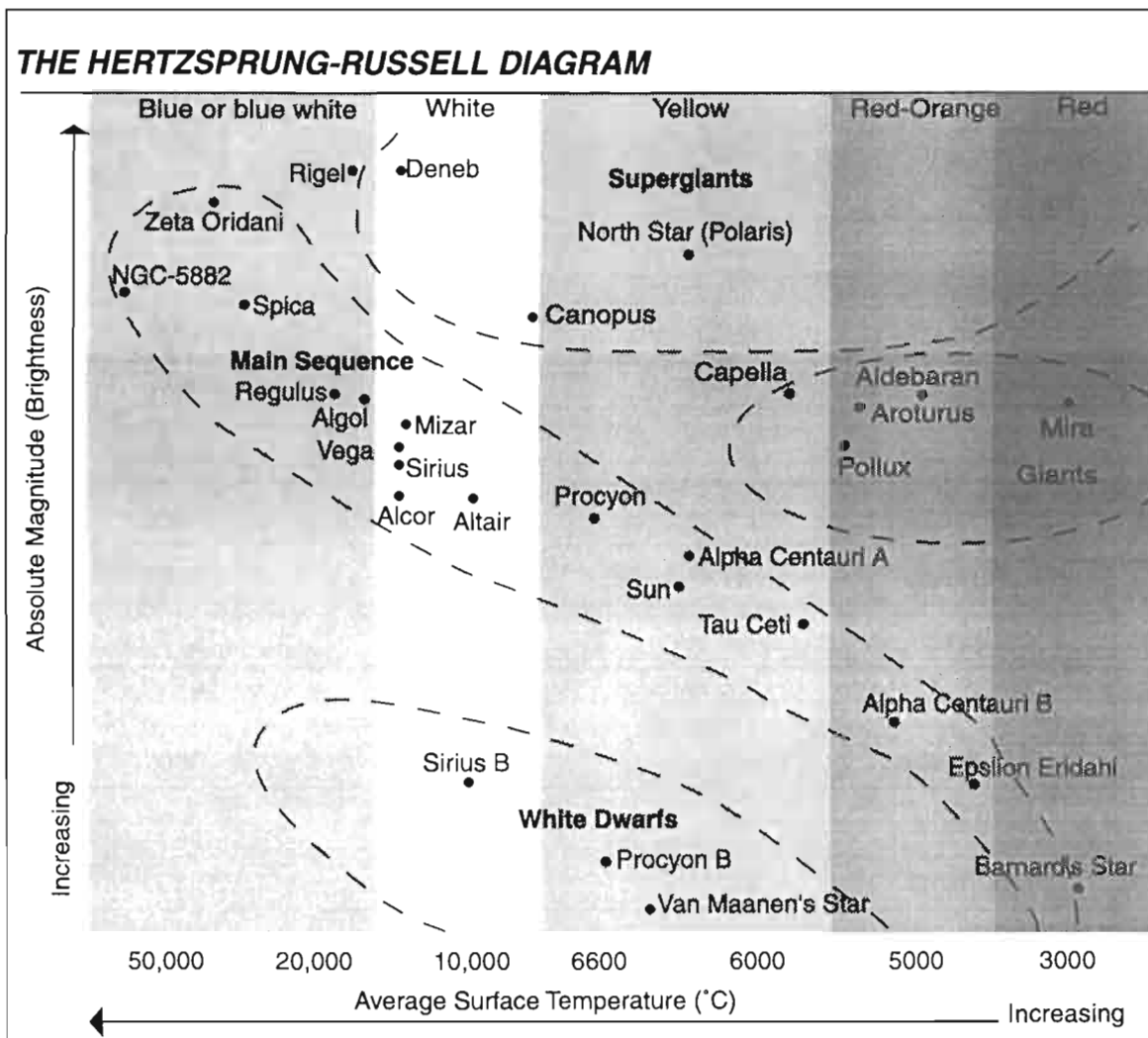
## Data for the Hertzsprung-Russell Diagram

(modified from Richard Moeschl, Exploring the Night Sky, p. 326)

Star Name	Spectral Type	Absolute Magnitude	Temperature (K)
Sirius	A1	1.4	9,500
Procyon	F5	2.8	6,500
Sun	G2	5.0	5,000
Barnard's Star	M5	13.2	2,600
Vega	A0	0.5	9,700
Van Maanen's Star	F0	14.2	5,800
Canopus	F0	-4.6	6,400
Deneb	A2	-7.1	9,400
Pollux	K0	1.0	4,100
Mintaka	O9.5	-5.1	21,000
Altair	A5	2.4	7,700
Regulus	B8	-0.7	13,000
Luyten 745-6	F0	14.3	5,900
Antares	M1	-3.0	2,700
Rigel	B8	-6.2	11,000
Betelgeuse	M2	-5.6	2,700
Aldebaron	K5	-0.5	3,500
Capella	G0	-0.5	5,000
Arcturus	K1	-0.0	3,900
Spica	B1.5	-2.2	19,500
Acrux	B1	-2.7	19,000
Formalhaut	A3	2.1	8,900
Rigel Kentaurus	G0	-0.5	5,800
Alnitak	O9.5	-5.9	23,000
Wolf 424 A	M6	14.4	2,500



# HERTZSPRUNG-RUSSELL DIAGRAM



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## UNIT 5:

# PATTERNS IN ASTRONOMY ALMANACS



## OVERVIEW FOR TEACHERS

### Unit Outline

#### Introduction

*The educational achievement is not to make the strange seem familiar, but to make the familiar seem strange. It is seeing the wonderful that lies hidden in what we take for granted that matters educationally.*

---Kiernan Egan, *Teaching as Story telling*

Nathaniel was thrilled with the challenges of mathematics. At the bright age of 14, Nat's brother introduced him to algebra. He couldn't sleep with excitement over the new world of using both letters and numbers to do his figuring. Soon Nathaniel was ready for navigation, a logical progression for someone indentured to the ship chandlery of Ropes and Hodges. It would only be a matter of time before Bowditch compiled a sailor's almanac, a sum-

mary of sun, moon, star and planet positions for each day of the year. An excerpt from *Carry on Mr. Bowditch* (p. 59) illustrates his passion:

*He was sixteen the summer he figured how to make an almanac. He felt a tingle go up his backbone. Just to think! A man could sit right here and figure out when the moon would rise every night, next month - or next year - or ten years from now! He could figure out the way the sun would act: he could figure ...*

*Ben Meeker shuffled into the chandlery one day. "What's that you're figuring on?*

*"An almanac for the years from 1789 to 1823."*

*Ben sniffed. "Do tell. And what's your almanac going to have in it?"*

*"Just the regular things: the sun's rising, setting, declination, amplitude, place in the ecliptic--"*

Nathaniel apparently wrote daily notes, much like a Captain's log, in a printed almanac dated 1789. The Bowditch family has preserved this almanac with its neatly printed inscriptions and drawings. Copies of a page with Nat's handwritten comments on the weather, sailing ships, and the comings and goings of Salem ship captains, are provided at the end of the Overview.

Even from the handwritten notes above, it is clear that the workings of the Universe were all a joyful matter of interest to Nathaniel Bowditch. Deciphering these workings were a matter of mathematics. The regularity of the sunrise and sunset, moonrise and moonset, as well as their ever changing placement in the night stars, were very much a part of every sailor's world. Sadly, these heavenly rhythms are lost to us in our fast-paced, modern lifestyles. The cyclic progression of constellations continue daily without our slightest appreciation or notice. Fortunately, students can become aware of these rhythms through many everyday sources. The daily newspaper, Farmer's Almanacs, and even the Internet offer daily almanacs for our diverse lifestyles.

In the following unit, entitled Student Almanacs, information is easily obtained by students to create individual almanacs. Students will rearrange and analyze data to uncover meaningful patterns from season to season. Students will also create graphs of changes over time, and summary tables for each of the almanac variables.

Students begin their discovery of the orderly movements of the sky through an investigation of sunrise and sunset data. To begin the journey, students graph the day-night cycle created by sunrise and sunset over a 7-day cycle. How predictable is the sunrise and the sunset? Students examine a large set

of data over many days to discover changing trends in both sunrise and sunset times. Numerical patterns are easily discovered which allow students to predict future sunrises and sunset times. Students quickly and easily extend their almanacs to cover at least two weeks ahead of the date of their current data. Their mathematical predictions according to the data trends can be verified by consulting the original almanacs.

But why are these trends occurring in the sunrise and sunset data? Can they tell us anything about the Earth and its travels around the sun? To answer these questions, students are assigned a research project. What is the yearly path of the Earth around the Sun? How does the Earth's  $23.5^\circ$  tilt on its axis effect this annual cycle? How does this tilt effect our seasons? Can we locate the seasonal path of the Sun's position anywhere on our Earth? Where? And finally, Why?

New data is introduced to the student almanac, the declination, or angle, of the sun at noon. The sun's declination at noon changes as the seasons move from winter to spring to summer to fall. As students uncover and then comprehend the sun's seasonal path between the Tropic of Capricorn, the Equator and the Tropic of Cancer, they will discover an explanation for the trends in their sunrise/sunset data. The final conclusion from this exercise is that students now take notice of the changing sun positions throughout the year. The changes in sunrise and sunset times are now understood to represent an effective mathematical diary of the Sun's seasonal progress across the face of the earth. Sailors, navigators, and astronomers alike use this knowledge to build and interpret many more complex relationships found in the sky.

A final note: the Student Almanac can be useful for many more data explorations. Additional areas of investigations include the movements of the planets in relation to the stars, the monthly pattern in moonrise and moonset, the relationship between tidal cycles and moon phases, and an exploration of Newton's Law of Gravity in relation to the orbiting moon, planets, and tides.

# Nathaniel Bowditch's Original Notes in an Almanac Dated 1789

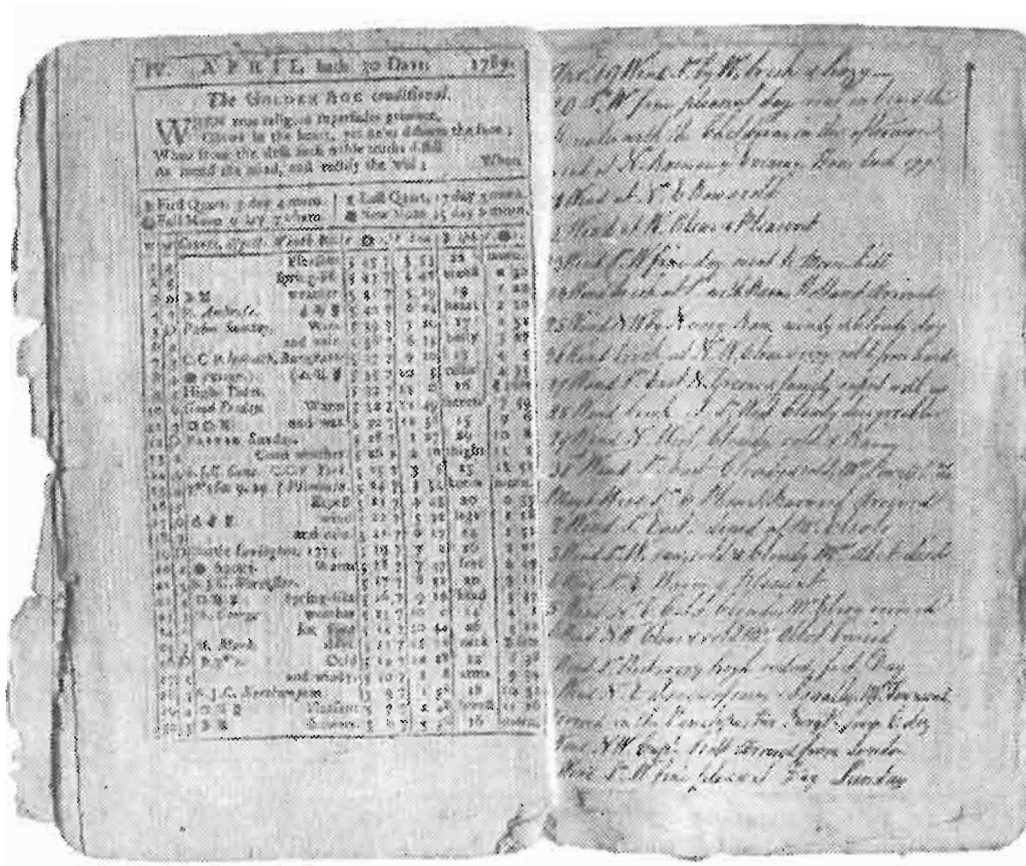
IV. APRIL hath 30 Days. 1789.									
The GOLDEN AGE conditional.									
<p>WHEN true religion supercedes grimace, Glow in the heart, yet ne'er distorts the face; When from the desk such noble truths distill As mend the mind, and rectify the will;      When</p>									
First Quart. 3 day 4 morn.					Last Quart. 17 day 3 morn.				
Full Moon 9 day 7 aftern.					New Moon 15 day 6 morn.				
W	C	A	P	R	I	L	S	E	X
1	4								
2	5								
3	6								
4	7								
5	8								
6	9								
7	10								
8	11								
9	12								
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19	22								
20	23								
21	24								
22	25								
23	26								
24	27								
25	28								
26	29								
27	30								
28									
29									
30									

Apr. 19 Wind S. by W. brisk & hazy -  
 20 S.W. fine pleasant day, went in boat to  
 Haverhill with the children in the afternoon  
 wind at N. rain in evening, Gen. died 11 p.m.  
 21 Wind at N. & Rain & cold  
 22 Wind at W. Clear & Pleasant  
 23 Wind S.W. fine day, went to Haverhill  
 24 Wind brisk at N. with heavy clouds & rain  
 25 Wind N.W. by N. very heavy, windy & cloudy day  
 26 Wind brisk at N.W. Clear & very cold from land  
 27 Wind S. East N. Green of family, super with us  
 28 Wind brisk at S. S. West & cloudy day, greenish  
 29 Wind N. West, Cloudy, cold & Rainy  
 30 Wind S. East, Greenish & cold, Mr. Powell m.  
 May 1 Wind S. & Pleasant, Green & Rainy  
 2 Wind S. East, died at Mr. Eliot's  
 3 Wind S.W. rain, cold & Cloudy Mr. Abbot died  
 4 Wind S. & Rain & Pleasant  
 5 Wind S. & Cold Cloudy, Mr. Gray arrived  
 6 Wind N.W. Clear & cold Mr. Abbot buried  
 7 Wind very high wind, fast Day  
 8 Wind S. & Green of rain, S. really, Mr. Townsend  
 9 Wind in the P.M. the first from C. d's  
 10 Wind S.W. Clear & Rain from London  
 11 Wind S. fine, pleasant & fine Sunday

Courtesy of Anna H. Bowditch Family



**Notes from a 1789 Almanac  
Containing Nathaniel Bowditch's  
Notes and Drawings.**



Courtesy of Anna H. Bowditch Family

**Objectives:**

- Students will compile common astronomical data from everyday sources such as *The Boston Globe* and *The Farmers Almanac*, and organize this data into tables entitled Student Almanac.
- The astronomical data in each almanac will include the following variables:
  - Sunrise time, sunset time
  - declination of sun
  - day of year
  - length of day
  - sun fast time

For future lessons:

- moonrise, moonset
- 1st and 2nd high tide
- 1st and 2nd low tide
- height of both high tides
- height of both low tides
- moon phase (8 phases)
- planet rise and set (Mercury, Venus, Mars, Jupiter, and Saturn).
- Students will use their sunrise/sunset data to mathematically illustrate the 24 hour rotation period of the earth on its axis, demonstrating the creation of day and night.
- Students will make predictions of future sunrise and sunset times using mathematical patterns uncovered in their daily almanac data.
- Students will relate these sunrise and sunset patterns to the yearly journey of the Sun across the face of the Earth.

**Additional objectives include:**

- Students will analyze and describe mathematical seasonal changes in sun rise and sunset at the winter/summer solstices and the spring (vernal)/fall (autumnal) equinoxes and will illustrate these seasonal differences using charts and graphs.
- Students will graph the changing positions of the planets as they move across a background of seasonal stars.
- Students will describe the regularly occurring patterns of Moon phases using both models of the Moon, Sun and Earth, as well as numerical illustrations (graphs, tables) of the changing pattern in moonrise, moonset, and changing phases of the moon.
- Students will use graphs to illustrate the relationship between the daily and seasonal tidal cycle and the phases of the Moon.
- Students will relate their understanding of the moon/tide cycles to the forces of gravity on the Earth's surface.
- Students will demonstrate their understanding of Sir Isaac Newton's Law of Gravity, by examining evidence that gravity is a force that produces an attraction between matter. Students will provide evidence that gravity pulls on or anywhere near the Earth toward the Earth's center and acts across space to hold the Moon in its orbit around the Earth and the planets in their orbits around the Sun.

**Skills:**

- Collecting data from outside sources.
- Organizing data into an appropriate table.
- Analyzing patterns and relationships between many variables.
- Illustrating scientific concepts using graphs.
- Summarizing trends and making mathematical predictions about future events from numerical patterns in the data.

**Vocabulary:**

- |                             |                      |                    |
|-----------------------------|----------------------|--------------------|
| • Sunrise time, sunset time | • declination of sun | • day of year      |
| • length of day             | • sun fast time      | • Tropic of Cancer |
| • Tropic of Capricorn       | • Solstice           | • Equinox          |

## **Frameworks connections:**

### **Science and Technology:**

#### **Strand 1: Inquiry, p. 28**

- Note/describe relevant details, patterns and relationships.
- Differentiate between questions that can be answered throughout direct investigation and those that cannot.
- Apply personal experience/knowledge to make predictions.
- Describe trends in data when patterns are not exact.
- Represent data and findings using tables, models, demonstrations and graphs.

#### **Strand 2: Domains of Science, pp. 77-78**

- Demonstrate an understanding that, like all planets and stars, the Earth is approximately spherical in shape. Use models to demonstrate how the rotation of the earth on its axis every 24 hours produces the night-and-day cycle.
- Observe and illustrate that planets change their positions against the background of stars.
- Observe and explain that the Earth has a natural satellite, the Moon, that circles the planet approximately every 29 days.
- Use models to describe how the motion of the Moon about Earth and the location of the Sun relative to Earth and its Moon are responsible for the regularly occurring patterns of Moon phases, eclipses and tides.
- Give evidence that gravity is a force that produces an attraction between matter. Gravity pulls on or anywhere near the Earth toward the Earth's center and acts across space to hold the Moon in its orbit around the Earth and the planets in their orbits around the Sun.

# Unit 5 Lesson Plans



## Lesson 1. What is an Almanac?

### Objectives:

- Students will compile useful astronomical data from everyday sources such as the Boston Globe and the Farmers Almanac and organize this data into a personal almanac.

### Skills:

- Collecting data from outside sources.

### Vocabulary:

- sunrise, sunset    • declination of sun    • day of year
- length of day    • sun fast time

### For future lessons:

- moonrise, moonset    • 1st and 2nd high tide
- 1st and 2nd low tide    • height of both high tides
- height of both low tides    • moon phases (8)
- planet rise and set (Mercury, Venus, Mars, Jupiter, and Saturn)

### Materials:

- *The Old Farmers Almanac*, by Robert B. Thomas, Dublin, NH 03444 - class set of 25 - 30 (1801 edition replicated)
- Other almanacs for further comparison
- Current *Boston Globe* Weather page - class set of 25 - 30
- What is an Almanac? - handout
- Creating Your Student Almanac - handout
- notebook
- paper for almanac title page
- colored pencils/markers
- ruler

### Procedure:

#### Class Challenge: (on board or overhead)

- What is an almanac? Brainstorm ideas, show several examples, etc.
- Distribute class copies of the Farmers Almanac.

- Title page (p. 1): What does it contain? When was it originally printed? What number is it? What else is on the cover?
- Contents (p. 2): What are the 2 major categories of articles? Which article would you like to read?
- Weather: What types of forecasts are in this almanac? What is the forecast for your region? Were the editors right in their forecast?
- Contents (p. 4): Review the Charts, Tables, and Miscellany: Which pages relate to your study of astronomy? What is the difference between astrology and astronomy?
- Astronomical Data (p. 4): Do you know all of these terms? What page contains information on how to use the almanac?

### **Individual Activity:**

- It's time to define a "true" almanac and to begin your own! What does your almanac state as a definition of a "true" almanac?
- Your personal almanac will need astronomical data from the left hand calendar pages. To simplify your search, we will begin with only a few variables at a time. Our first almanac chart will need only information about the sun. Which columns should you select?
- Rises, Sets, Length of Day, Sun Fast, Declination of Sun: All of these relate to the sun. Review the explanations of these terms before we go on.
- You will also need to know the information in the first column, Day of the Year. Create your own column format for Date (ex: 12/10/00, or Dec. 3, 2000)
- Find today's date and enter all of the data you selected for the sun.
- What units are these variables? This is very important to know!
- Copy 7 days of sun data into your almanac. We will begin searching for patterns tomorrow.

### **Homework Suggestion:**

- Create your own colorful almanac cover at home.



# WHAT IS AN ALMANAC?

## Class Challenge:

1. What is an almanac? First, brainstorm ideas with your classmates.
2. Let's investigate by looking at a copy of *The Old Farmers Almanac*

### Title page:

- What does it contain?
- When was it originally printed?
- What number is it?
- What else is on the cover?

### Contents:

- What are the major categories of articles?
- Which articles would you find interesting to read?

### Weather:

- What types of forecasts are in this almanac?
- What is the forecast for your region?
- Were the editors right in their forecast?

**Contents:**

- Charts, Tables, and Miscellany:
- Do any pages relate to your study of astronomy?
- What is the difference between astrology and astronomy?

**Astronomical Data:**

- Do you know all of these terms?
- List the terms that you do not know:
- What page contains information on how to use this almanac?





# CREATING YOUR STUDENT ALMANAC

## Individual Activity:

1. It's time to define a "true" almanac and to begin your own!
  - In the section titled, "How to Use This Almanac" or "Explanation of the Calendar Pages", what pages are the heart of the almanac?
  - Describe the contents of a "true almanac".
2. Your personal, student almanac will need astronomical data from the calendar pages, the left-hand side. To simplify your search, we will begin with only a few variables at a time.
  - Our first almanac chart will need only information about the sun. Find the calendar page for November. Which columns refer to the sun?
  - Use the almanac's Glossary and the explanations on the page titled Left-Hand Calendar Pages to define the following terms:

Sunrise/Sunset

Length of Day

Sun Fast

Declination of Sun

- All of these terms relate to the sun.

Skim the Glossary pages for other terms you might know.

3. Create a table for your almanac of the sun:

- Place a title at the top, along with your name and class period.

Enter:

- Column 1: Day of the Year
- Column 2: Day of Month:
- Column 3: Day of Week
- Column 4: Rises (h. m.)
- Column 5: Sets (h. m.)
- Column 6: Length of Day (h. m.)
- Column 7: Sun Fast (m.)
- Column 8: Declination of Sun (degrees minutes)

4. If you are handy with creating spreadsheets using EXCEL or other software, this is an excellent opportunity to practice your skills!

5. Copy 7 days of data, starting with today, into your almanac. We will begin searching for patterns in the next activity.

6. Homework suggestion: Create a colorful almanac cover at home.



## Lesson 2. What Can We Do With All of These Numbers?

### Objectives:

- Students will use their sun data to mathematically illustrate the 24 hour rotation period of the earth on its axis, demonstrating the creation of day and night.
- Students will make predictions of future sunrise and sunset times using mathematical patterns uncovered in their daily almanac data.

### Skills:

- Collecting data from outside sources.
- Analyzing patterns and relationships between many variables.
- Illustrating scientific concepts using graphs and tables.
- Summarizing trends and making predictions about future events from patterns in the data.

### Vocabulary:

- |                      |               |                 |
|----------------------|---------------|-----------------|
| • sunrise time       | • sunset time | • day length    |
| • declination of sun | • day of year | • sun fast time |

### Materials:

- |                          |                                        |
|--------------------------|----------------------------------------|
| • Student almanac cover  | • The Mathematics of the Sun - handout |
| • centimeter graph paper | • pencil                               |
| • ruler                  |                                        |

### Procedure:

#### Class Challenge:

- How long is 1 day on our Earth?
- Mathematically define one Earth day for your classmates. Illustrate your definition with a chart and a graph.
- Why do we have a "day" and a "night"?
- Mathematically demonstrate the creation of day and night. Illustrate your proof with a chart and a graph.

**Activity:**

- Can you predict the sunrise without consulting your almanac?
- First, is there a pattern in the daily changes in sunrise for the 5 to 7 days you copied into your almanac?
- Subtract each day's sunrise time from the previous day to see if the changes are consistent from one day to the next.
- How can this information help you predict the sunrise for the next day you do not have data for?
- Without consulting *The Farmer's Almanac*, calculate the sun rise times for the 7 days after your own almanac data.
- Now, recheck *The Farmer's Almanac* for your predicted dates. How accurate were you? Explain your results.
- Now, repeat this exercise for sunset times. Determine the difference in daily times and use this difference to predict the 7 days after your own data.
- Recheck *The Farmer's Almanac* for your predicted sunset times. How accurate were you? Explain your results.

**Analysis:**

- What is a prediction in science or mathematics?
- Why are you able to predict sunrise and sunset times?
- What can these daily changes in sunrise and sunset tell us about our Sun and our Earth?
- Does the Earth's tilt, moving to different locations in the yearly revolution around the Sun, determine our different seasons?

**Independent Research:**

- To answer these questions you must research the relationship between the Earth's yearly revolution around the Sun and the Earth's constant 23.5° tilt on its axis.
- Use as many resources as you can find to help you illustrate:
  - the tilt of the Earth on its axis as the Earth revolves around the sun.
  - the position of the Earth and the Sun at each of the four seasons of the year

- find both on a globe and then draw, the location of the Tropic of Capricorn, the Equator, and the Tropic of Cancer.
- find both on a globe and then draw, the location of the Sun at the winter solstice, spring equinox, summer solstice, and fall equinox.





# THE MATHEMATICS OF THE SUN

## Class Challenge:

1. How long is 1 day/night cycle on our Earth?

- Is it the same length of time for every complete cycle or does it change with the seasons? Does it change from one year to the next?

- Use your almanac to research the questions above, then mathematically define one complete Earth day/night cycle. Illustrate the class definition with a chart and a graph.

2. Why do we have a "day" (daylight) and a "night" (darkness)?

- Use your almanac to research the question, then mathematically demonstrate the creation of day and night. Illustrate your class proof with a chart and a graph.

- Is the amount of daylight and darkness the same throughout the year? Why?

### Individual Activity:

- Can you predict the sunrise time without consulting your almanac?
- First, determine if there is a pattern in the daily sunrise times you entered into your almanac:
- If you see a pattern, check your hunch by subtracting the sun rise time from each previous day to see if the changes are consistent from one day to the next. Show your work in a table below:
- Can this information help you predict the sunrise time for the 8th day?
- Without consulting *The Farmer's Almanac*, calculate the sunrise times for the next 7 days following your own almanac data. Show your work in a table below:
- Now, recheck *The Farmer's Almanac* for your predicted dates. How accurate were you? Explain your results.



- Repeat this exercise for sunset times. Determine the difference in daily times and use this difference to predict the 7 days after your own data. Show your work in a table below:
- Recheck *The Farmer's Almanac* for your predicted sunset times. How accurate were you? Explain your results.

### Analysis:

- What is a prediction in science or mathematics?
- Why are you able to predict sunrise and sunset times?
- Did the length of day change with changing sunrise and sunset? Using your original 7 days of sun data, create a graph of length of day (vertical axis) versus date (horizontal axis). Explain your results below:
- Select a week that is 4 months ahead of your current date. Did the same patterns in sunrise and sunset, and in length of day, occur? Create tables and graphs to illustrate your results. Explain your results below:

**Questions to think about:**

- What can these daily changes in sunrise and sunset tell us about the location of our Sun in respect to the Earth?
- Does the Earth's tilt, moving to different locations in the yearly revolution around the Sun, have an effect on your sunrise, sun set, and length of day results?

**Independent Research:**

- To answer these questions you must research the relationship between the Earth's yearly revolution around the Sun and the Earth's constant  $23.5^\circ$  tilt on its axis.
- Use as many resources as you can find to help you illustrate:
  - the tilt of the Earth on its axis as the Earth revolves around the sun.
  - the position of the Earth and the Sun at each of the four seasons of the year
  - find both on a globe and then draw, the location of the Tropic of Capricorn, the Equator, and the Tropic of Cancer.
  - find both on a globe and then draw, the location of the Sun at the winter solstice, spring equinox, summer solstice, and fall equinox.



## Lesson 3: A Diary of the Sun.

### Objectives:

- Students will relate the sunrise and sunset patterns in their almanac data to the seasonal journey of the Sun across the face of the Earth.

### Skills:

- Illustrating scientific concepts using graphs, tables, and models.

### Materials:

- Sunrise/sunset predictions
- Research results from Lesson 2
- The Sun's Diary - handout
- Map of the world with latitude and longitude clearly marked
- centimeter graph paper
- pencil
- ruler

### Procedure:

#### 1. Class Challenge(on board or overhead):

Answer the following questions from your research:

- How does the Earth's  $23.5^\circ$  tilt, moving to different locations in the yearly revolution around the Sun, determine our different seasons?
- Where are the Sun's rays shining directly overhead at the winter solstice? spring equinox? summer solstice? fall equinox?
- Is your variable, Declination of Sun, related to this changing position of the Sun through the seasons?
- Can you use this variable and your research knowledge to explain why your sunrise data and your sunset data are changing daily?

#### 2. Activity:

- Create a graph with the variables Day of Year (corresponding to your 14 sunrise dates) on the horizontal axis and Declination of the Sun (degrees and minutes) on the vertical axis.
- Graph these points for each day of your almanac data.
- Next, on your globe, point to the degrees latitude that match as closely as possible the same degrees as your Declination of the Sun on your graph.

- Explain the relationship, now, between your changing sunrise and sunset times with the change in the Sun's declination over the same time period.

### **3. Group Analysis:**

- Find the dates of the two solstices and the two equinoxes. Examine the changing Declination of the Sun for each of these dates.
- Each group member should now create a graph of the Day of Year and Declination of the Sun for one week before and one week after his/her assigned equinox or solstice.
- Compare the graphs within the group.

### **4. Class Analysis (on board or overhead)**

- Describe the seasons according to your graphs of the Declination of the Sun.
- Relate these graphs to the location of the Earth as well as the tilt of the Earth in its journey around the Sun.
- Can you create a mathematical description of the sun's angled rays as they travel seasonally from the Tropic of Capricorn to the Tropic of Cancer?
- Summarize the reason for the seasons. Include the revolution of the Earth around the Sun, the Earth's  $23.5^\circ$  tilt on its axis, and the changing declination of the Sun.



# THE SUN'S DIARY

## 1. Class Challenge:

Answer the following questions from your research in Lesson 2:

- How does the Earth's  $23.5^\circ$  tilt, moving to different locations in the yearly revolution around the Sun, determine our different seasons? Illustrate and explain below:

- Illustrate and explain where the Sun's rays are shining directly overhead at noon on the winter solstice, the spring equinox, the summer solstice, and the fall equinox:

## 2. New Challenge:

Is your almanac variable, Declination of Sun, related to the changing position of the Sun through the seasons? Explain below:

### 3. Activity:

- Use the variable, Declination of the Sun, and your research knowledge, to explain why your sunrise data and your sunset data are changing daily:
- Create a graph with the variables, Day of Year, (corresponding to your 14 sunrise dates) on the horizontal axis and Declination of the Sun (degrees and minutes) on the vertical axis.
- Graph these points for each day of your almanac data.
- Next, on your globe, locate the degrees latitude that match as closely as possible the same degrees as your Declination of the Sun on your graph.
- Explain the relationship, now, between your changing sunrise and sunset times with the change in the Sun's declination over the same time period:

### 4. Group Analysis:

- Open your almanacs to the dates of the two solstices and the two equinoxes. Examine the changing Declination of the Sun for each of these dates.
- Each group member should be assigned one of these dates. Create a graph of the Day of Year and Declination of the Sun beginning one week before and ending one week after your assigned equinox or solstice.
- Compare/contrast the graphs within the group:

**Winter solstice declination-**

Compare

Contrast

**Spring equinox declination-**

Compare

Contrast

**Summer solstice declination**

Compare

Contrast

**Fall equinox declination:**

Compare

Contrast

**5. Class Analysis:**

- Describe the seasons according to your new graphs of the Declination of the Sun at different dates:
- Relate these graphs to the location and tilt of the Earth in its seasonal journey around the Sun.
- Create a mathematical description or diary of the sun's angled rays as they travel seasonally from the Tropic of Capricorn to the Tropic of Cancer and back again. Use your almanac data ( sunrise, sunset, declination of sun, length of day) in your description.



## UNIT 6

# PATTERNS IN ARCHITECTURE



*Nathaniel Bowditch House, c. 1890, Essex St., Salem, MA  
Courtesy of, copyright of the Peabody Essex Museum, Salem, MA*



## OVERVIEW FOR TEACHERS

### Unit Outline

#### Introduction:

*Home for Bowditch [in the early 1800s] became the beautifully proportioned Federal mansion on Chestnut Street [now 12 Chestnut Street] known as the Hodges House. The building was a two-family house, three stories high, its symmetrically spaced windows with green shutters. (Yankee Stargazer, 144)*

Architecture is the marriage of mathematics and art. General mathematical skills such as observation are as necessary as the more specific mathematical skills of geometry, symmetry, and congruence. An understanding of architecture also demands the visual and aesthetic talents of the artist.

In this unit, students will examine four basic types of architecture popular in New England, and specifically, Salem, in the early 1800's. Students will apply the mathematical concepts of symmetry, congruence, and proportion as they analyze the common New England architectural designs. The analyses and applications will result in a greater appreciation of the beauty and durability of the regional architecture.

**Objectives:**

- Students will identify the following plane shapes: circle, semicircle, quarter-circle, ellipse (oval), square, rectangle, rhombus, trapezoid, triangle, pentagon, hexagon, octagon, quadrilateral
- Students will determine whether various shapes are congruent or not.
- Students will identify lines of symmetry of various plane figures.
- Students will analyze four basic styles of American architecture, identifying characteristic shapes, symmetry, and congruence
- Students will determine several solutions to dividing a 10-by-20 unit rectangle into two congruent parts.

**Skills:**

- Students will be able to recognize geometric shapes.
- Students will comprehend the concepts of symmetry and congruence.
- Students will apply their skills to real-life situations, such as the architecture of Salem.

**Vocabulary:**

- |                  |                 |              |
|------------------|-----------------|--------------|
| • circle         | • square        | • triangle   |
| • semicircle     | • rectangle     | • pentagon   |
| • quarter-circle | • rhombus       | • hexagon    |
| • ellipse        | • trapezoid     | • octagon    |
| • symmetry       | • quadrilateral | • congruence |

**Frameworks Connections:****Mathematics****Strand 3: Geometry and Measurement****Standard 3.3: Geometry (p. 75)**

1. Identify, describe, compare, classify geometric figures.
2. Explore and describe properties of planes.
3. Visualize and draw geometric figures.
4. Explore and describe transformations of geometric figures.
5. Apply geometric properties and relationships.

# Unit 6 Lesson Plans



## Lesson 1: Basic Plane Shapes

### Objectives:

- Students will define congruence and give examples.
- Students will identify any and all lines of symmetry in various shapes.

### Skills:

- Students will be able to identify the following plane shapes: circle, semicircle, quarter-circle, ellipse (oval), square, rectangle, rhombus, trapezoid, triangle, pentagon, hexagon, octagon, quadrilateral.
- Students will know how to draw any and all lines of symmetry in various plane shapes.

### Vocabulary:

- |                  |                 |              |
|------------------|-----------------|--------------|
| • circles        | • square        | • triangle   |
| • semicircle     | • rectangle     | • pentagon   |
| • quarter-circle | • rhombus       | • hexagon    |
| • ellipse        | • trapezoid     | • octagon    |
| • symmetry       | • quadrilateral | • congruence |

### Procedure:

1. Distribute worksheet "Getting Into Shapes".
2. Have students discuss the characteristics of the various shapes. What differentiates a square from a rectangle? A rhombus from a square?
3. Have students look at the examples of congruence. Why are all triangles not congruent?
4. Have the students look at the examples of symmetry. Note that some shapes have more than one line of symmetry; some have none.
5. Distribute worksheet "Which Pairs Are Congruent?" Discuss why some shapes are congruent (they are the same shape and same size even though they may be rotated differently) and some are not congruent (they may be the same shape but not the same size).

6. Distribute worksheet "Symmetry" and have students draw any and all lines of symmetry.

**Handouts:**

Worksheets:

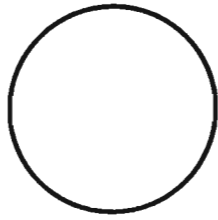
"Getting Into Shapes" (2 pp.)

"Which Pairs Are Congruent?" (2 pp.)

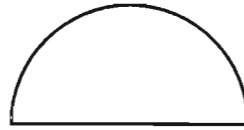
"Symmetry" (2 pp.)



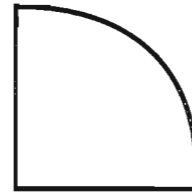
# GETTING INTO SHAPES



Circle



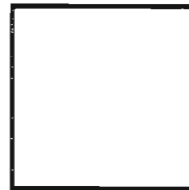
Semicircle



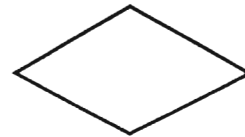
Quarter-circle



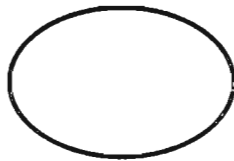
Rectangle



Square



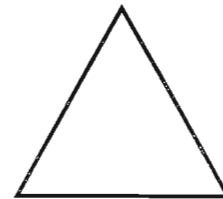
Rhombus



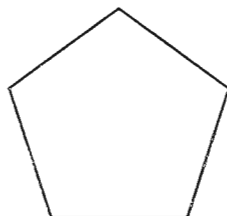
Ellipse



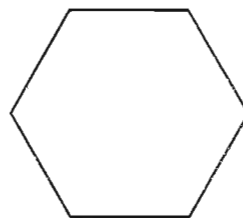
Trapezoid



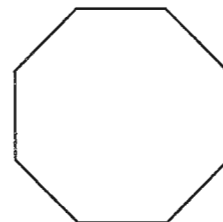
Triangle



Pentagon



Hexagon



Octagon

All of the figures above are *plane* figures. That means that they have *length* and *width*. They are said to be *two-dimensional*.

A *quadrilateral* is a closed plane figure made up of four line segments. Which of the figures above are quadrilaterals?

How many sides does a pentagon have? How many sides does a hexagon have? How many sides does an octagon have?

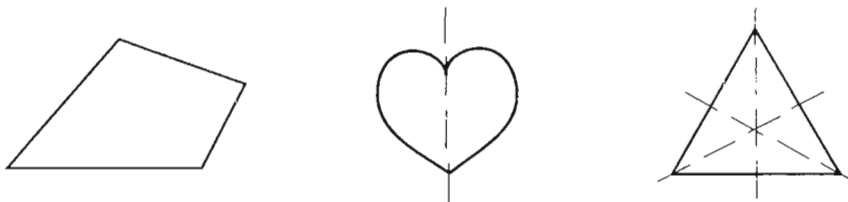
Plane figures may have other properties, including *congruence* and *symmetry*. Two plane figures are congruent if they are the same size **and** the same shape. Look at the pair of figures below:



They are not congruent. They are the same shape (squares, in this case), but not the same size. The two figures below are congruent, because they are the same shape and the same size, even though they are not 'pointing' the same way.



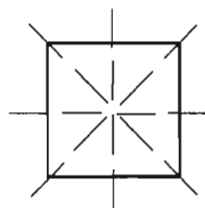
A plane figure has symmetry if a line can be drawn through the figure in such a way that if the figure were folded in half along the line of symmetry, the halves would match exactly. Some figures have more than one line of symmetry, some have none. Study the figures below for symmetry.



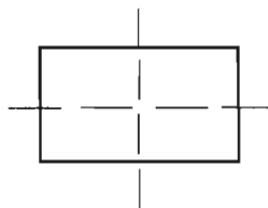
No lines of symmetry

1 line of symmetry

3 lines of symmetry



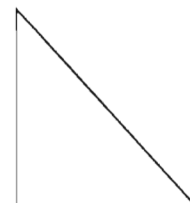
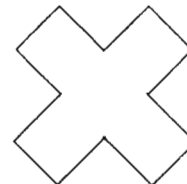
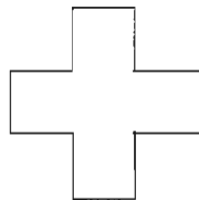
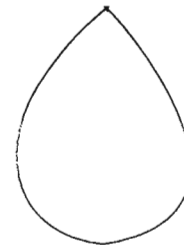
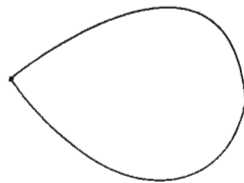
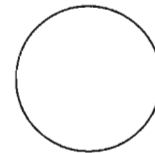
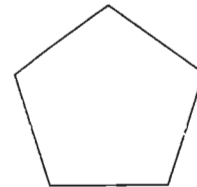
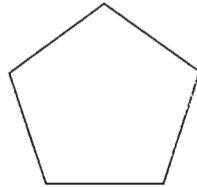
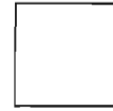
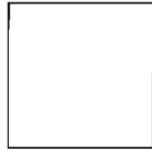
4 lines of symmetry



2 lines of symmetry

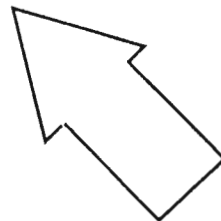
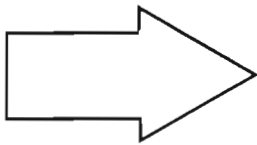
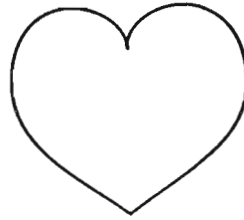


## WHICH PAIRS ARE CONGRUENT?





## WHICH PAIRS ARE CONGRUENT?



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# SYMMETRY

Each of the capital letters below have at least one line of symmetry. Sketch the lines of symmetry. Identify which letter has an infinite number of symmetry lines.

D

O

T

V

E

X

C

U

B

H

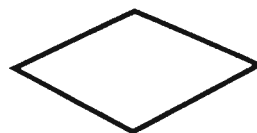
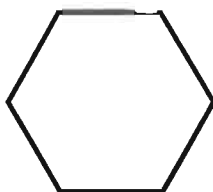
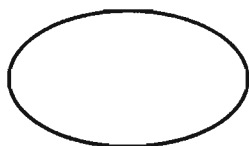
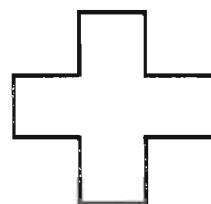
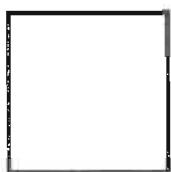
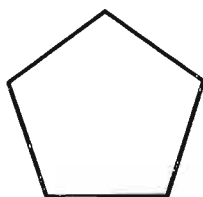
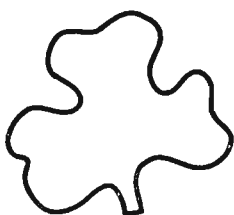
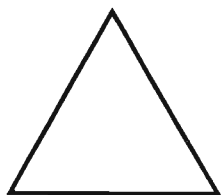
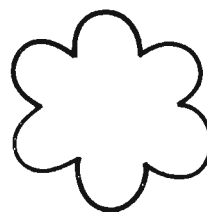
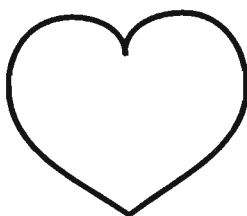
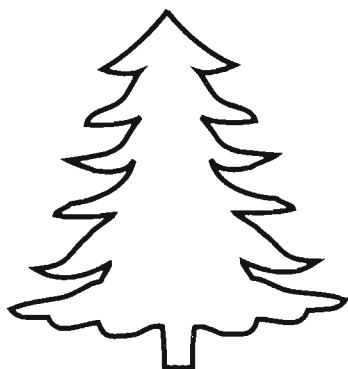
A

M



# SYMMETRY

Sketch the lines of symmetry in the figures below. Remember that some figures will have more than one line of symmetry.



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## Lesson 2: Math and Architecture

### Objectives:

- Students will analyze four basic styles of architecture and identify the characteristic shapes featured in each style.
- Students will identify the architectural styles and determine whether they contain symmetry or not.

### Skills:

- Students will understand how to create multiple solutions to a single problem.

### Vocabulary:

- New England Colonial
- Georgian Style
- Federalist
- Victorian

### Materials:

- worksheets

### Procedure:

1. Distribute worksheet "Math and Architecture" and the line drawings of four basic architectural house styles (New England Colonial, Georgian, Federalist, Victorian).
2. Students will examine each style for symmetry and congruence.
3. Identify the shapes featured in each style.
4. Distribute worksheet "Extra Challenge" and have students think of as many ways as possible to divide a 10-by-20 rectangle into two congruent parts.

### Handouts:

#### Worksheets:

"Math and Architecture"

"Extra Challenge"

## Math and Architecture Answers

On the next four pages are drawings of four different kinds of architecture seen commonly in New England and Salem. Study the drawings and then answer the questions below:

### New England Colonial

Do you see symmetry? (no)

Are the window congruent? (no)

What is the overall shape of the house (minus the chimney)? (pentagon)

What shape are the window panes? (rhombus)

What shape is the door? (rectangle)

### Georgian house

Do you see symmetry? (yes)

Are the windows all congruent (yes)

What are the shapes of the following areas?

A. (trapezoid)

B. (rectangle)

C. (triangle)

### Federalist house

Do you see symmetry? (no-quite a bit, but not completely---because of chimney, side portico)

Are all the windows congruent? (no)

What are the shapes of the following areas?

D. (semicircle)

E. (trapezoid)

### Victorian House

Do you see symmetry (no)

Are all the window congruent? (no)

What are the shapes of the following areas?

F. (triangle)

G. (rhombus)

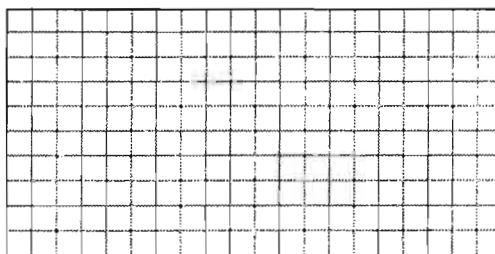
H. (ellipse)

I. (rectangle)

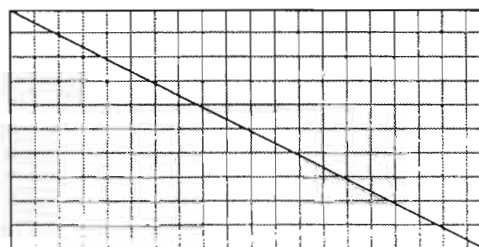
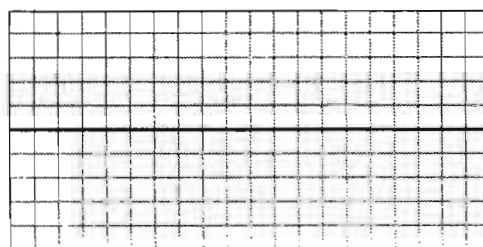
J. (quarter-circles)

## Extra Challenge Answers

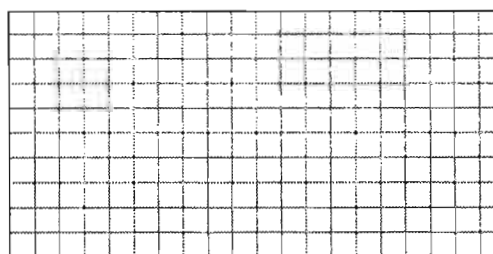
Let's pretend the figure below is a piece of property which you are to share equally with your neighbor. What is the area of the property? (120 square units) How many squares units do each of you receive? (60 square units)



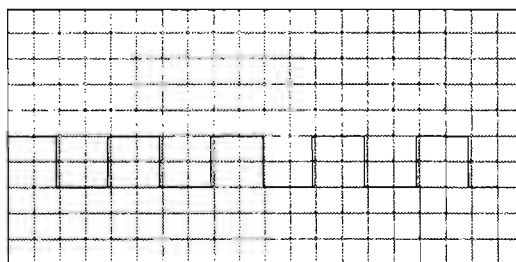
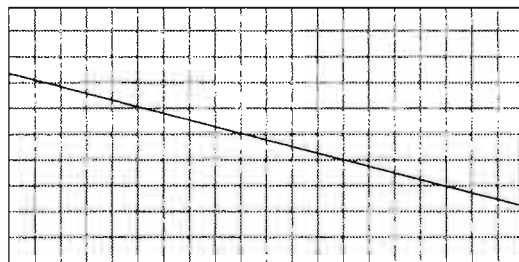
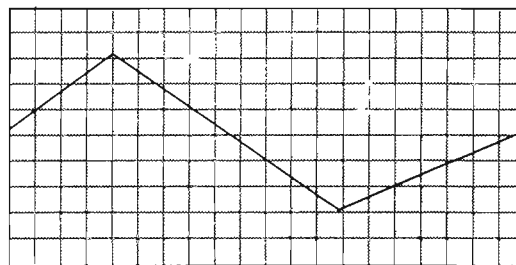
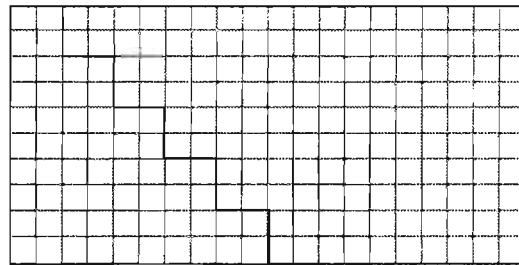
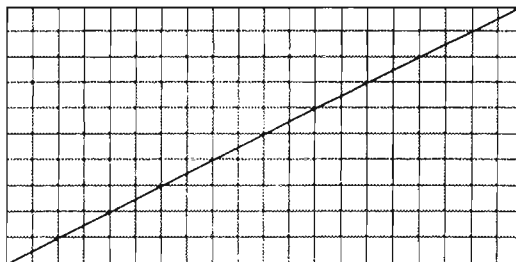
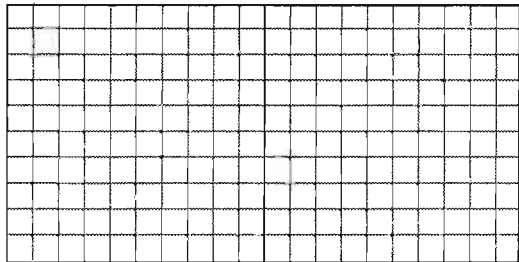
Here's the challenging part: how do you divide the property so each of you get congruent shares of the land? A couple of obvious ways are shown below.



Your challenge is to find other ways to divide the parcel of land into two congruent parts. Use your imagination.

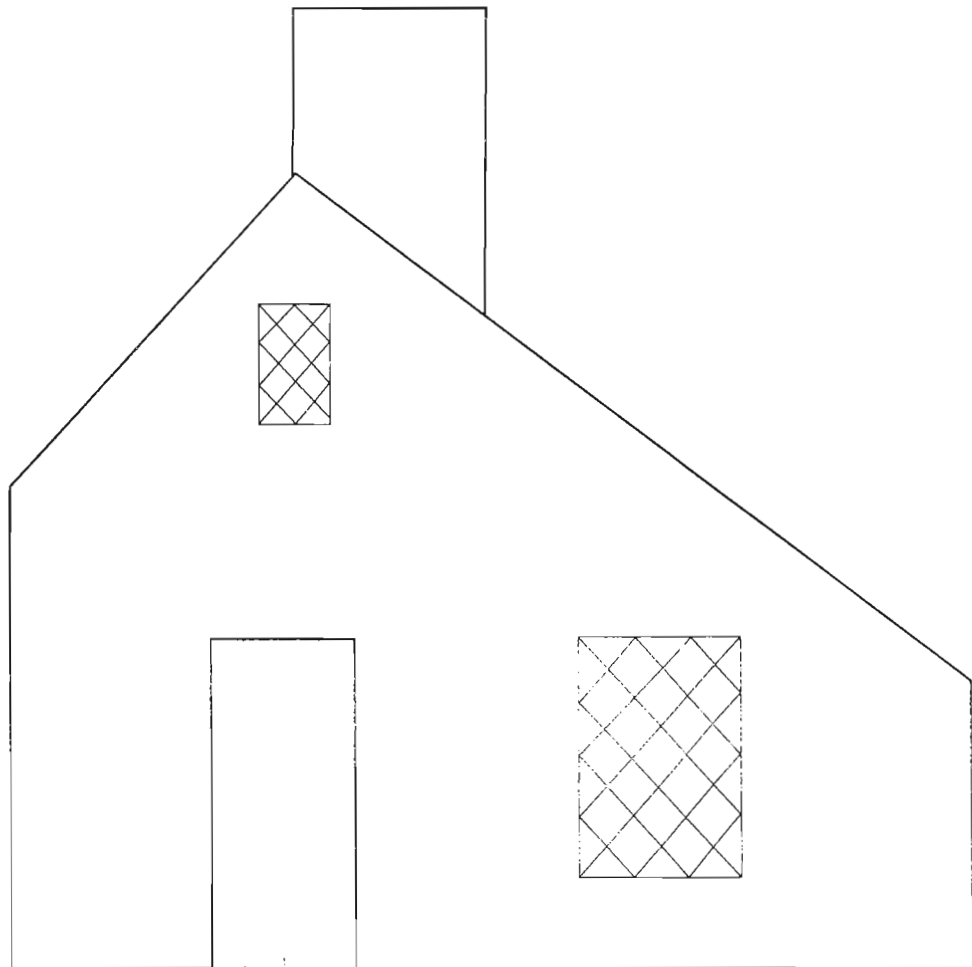


## Extra Possible Chart Answers





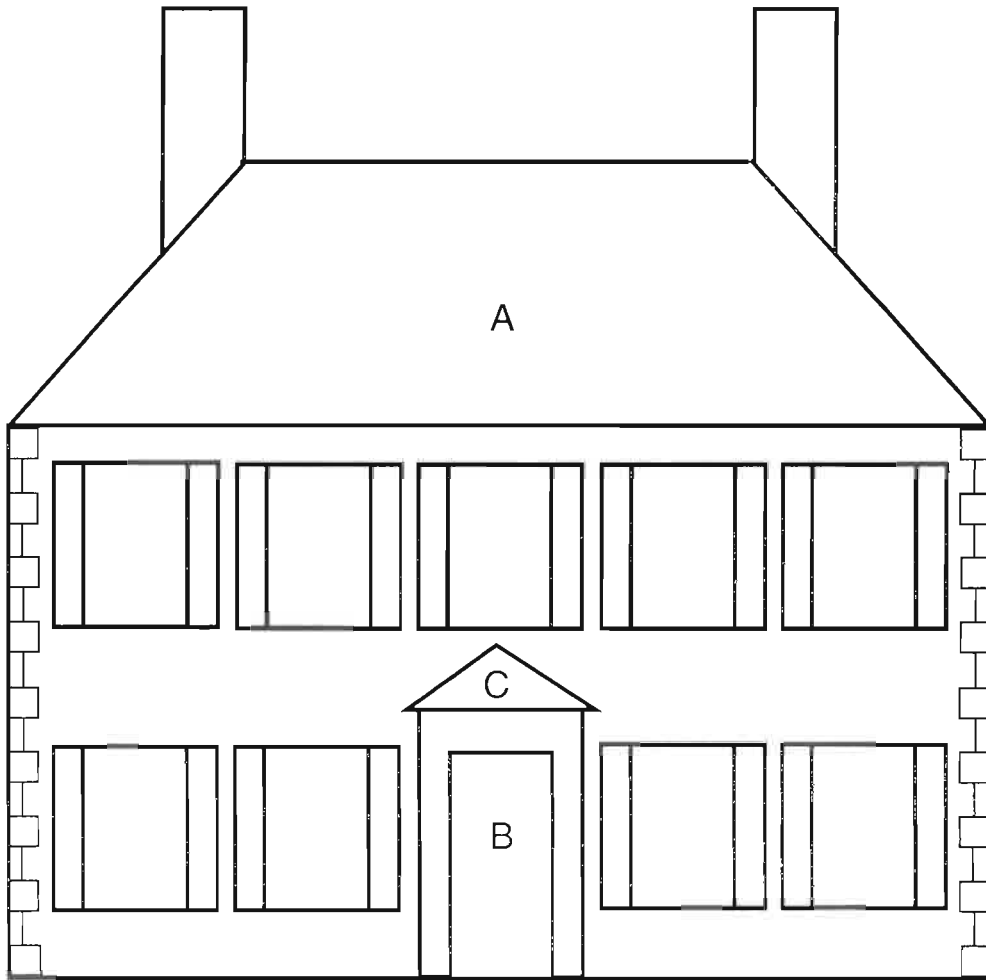
# The New England Colonial



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## Georgian 1735-1790

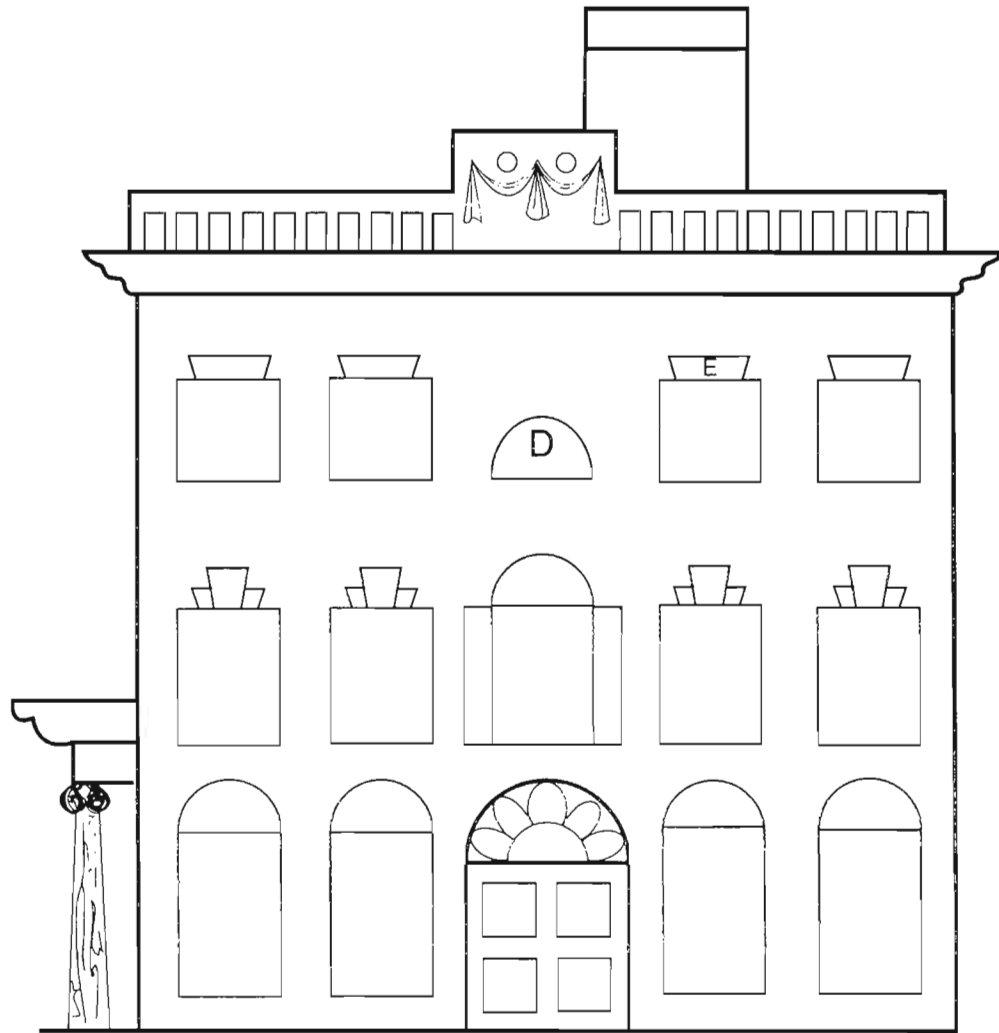


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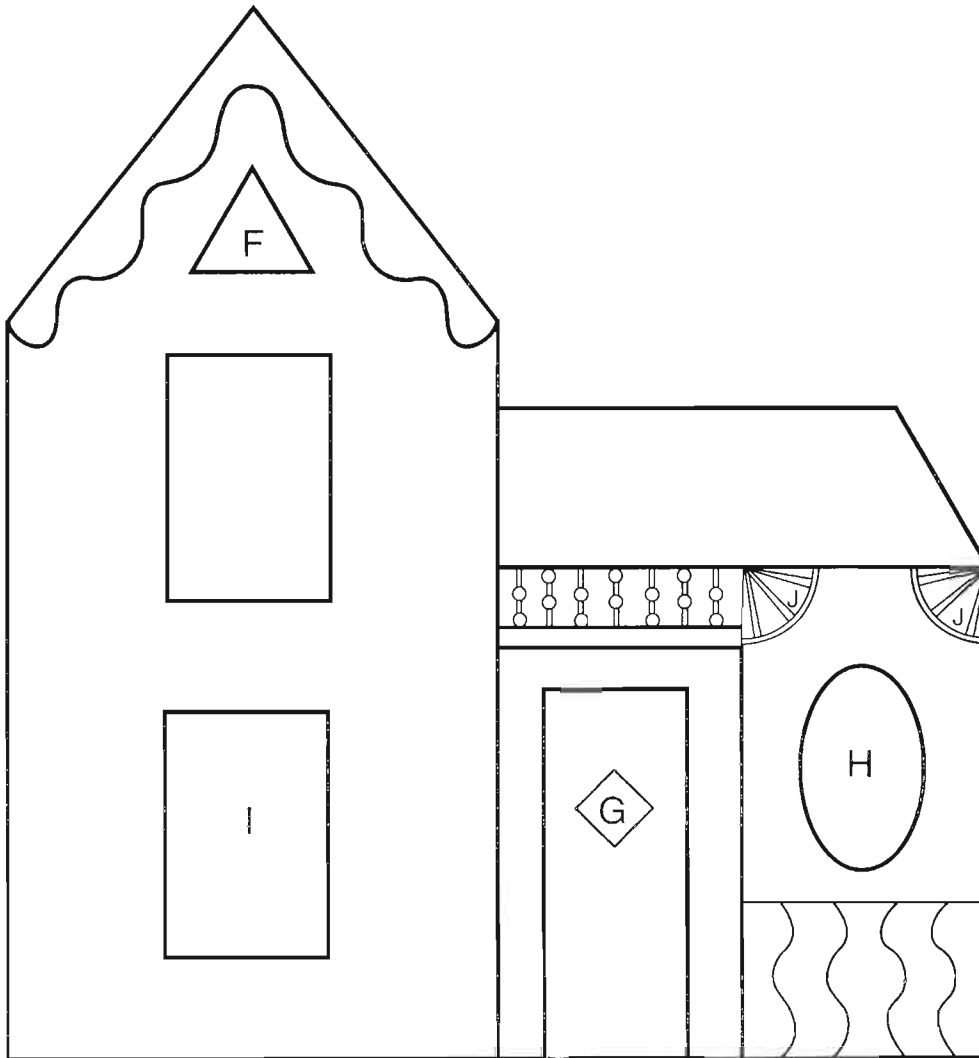
# Federalist 1790-1820



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## Victorian 1840-1900

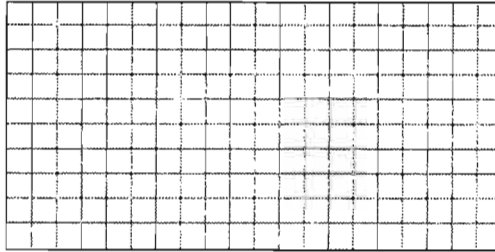


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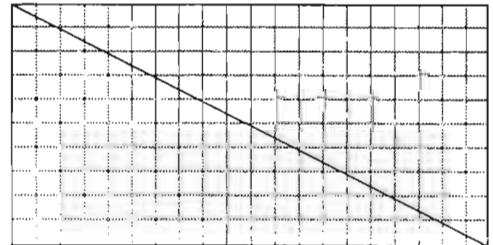
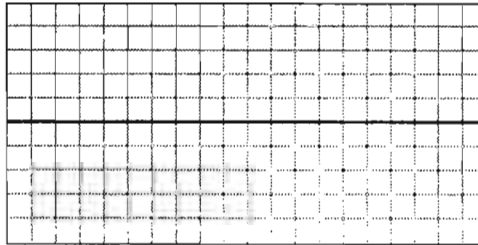


## EXTRA CHALLENGE

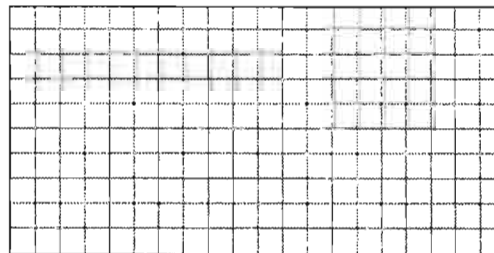
Let's pretend the figure below is a piece of property which you are to share equally with your neighbor. What is the area of the property? How many squares units do each of you receive?



Here's the challenging part: how do you divide the property so each of you get congruent shares of the land? A couple of obvious ways are shown below.



Your challenge is to find other ways to divide the parcel of land into two congruent parts. Use your imagination.





# MATH AND ARCHITECTURE

On the next four pages are drawings of four different kinds of architecture seen commonly in New England and Salem. Study the drawings and then answer the questions below:

## **New England Colonial**

Do you see symmetry?

Are the window congruent?

What is the overall shape of the house (minus the chimney)?

What shape are the window panes?

What shape is the door?

## **Georgian house**

Do you see symmetry?

Are the windows all congruent

What are the shapes of the following areas?

A.

B.

C.

## **Federalist house**

Do you see symmetry?

Are all the windows congruent?

What are the shapes of the following areas?

D.

E.

## **Victorian House**

Do you see symmetry

Are all the window congruent?

What are the shapes of the following areas?

F.

G.

H.

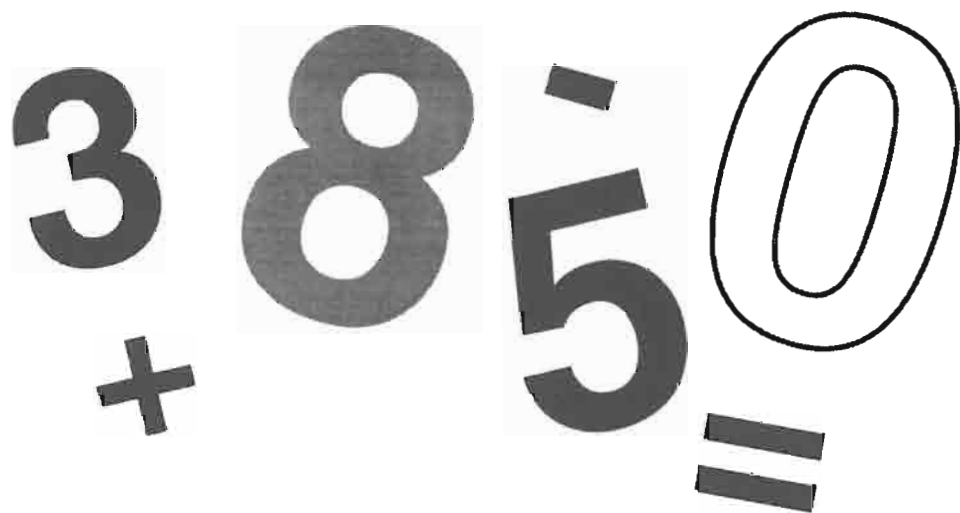
I.

J.

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## UNIT 7

# MATH SIMPLIFICATION


$$3 + 85 = 0$$



## OVERVIEW FOR TEACHERS

### Unit Outline

#### Introduction:

*...make the mathematics simpler. Plain arithmetic instead of calculus or spherical trigonometry and whatever else it is you use. That's for astronomers, not for sailors. Express it all in simple tables and formulas...*

(Captain Henry Prince to Nathaniel Bowditch in *To Steer by the Stars*, p. 112)

Simplification---solving a similar, but simpler problem, is a basic problem-solving strategy. Nathaniel Bowditch learned as a young man that all the fancy formulas in the world were worthless if the people who needed to use them could not comprehend them. He, therefore, brought the mathematics used in navigation down to a level that sailors with only the most primitive arithmetic skills could understand and apply.

Nathaniel was a natural teacher. Anxious for his pupils to understand the complexities of navigation or astronomy, Nat persevered in creating always a simpler explanation. Horace Mann, the renowned educator later in the 19th century, explains Bowditch's aptness to teach in his wonderful book, *The Art of Teaching*:

*...as a dramatic writer throws himself successively into the character of the drama he is composing that he may express the ideas and emotions peculiar to each other, so the mind of a teacher should migrate, as it were, into those of his pupils to discover what they know and feel and need; and then supply from his own stock what they require, he should reduce it to such a form and bring it within such a distance that they can reach it and seize it and appropriate it.*  
Horace Mann, *The Aptness of Teaching*, p. 16-17.

### **Objectives:**

- Students will calculate the area of rectangles by counting the squares on graph paper and by applying the simple formula  $A = L \times W$ .
- Students will calculate the area of odd shapes by breaking the figure into rectangles, finding the area of the individual rectangles and adding them up to find the total area.
- Students will use creativity in problem-solving by brainstorming with a team and comparing their solutions with others.
- Students will be challenged to solve a problem regarding a land dispute that Bowditch was called on to solve. Students will find the areas of rectangles drawn on graph paper by counting the unit squares inside the rectangle.
- Students will compare the populations and areas of twelve North Shore communities and calculate the population density of each community. They will represent this information visually on graph paper.
- Students will compare the average house prices of the same twelve communities and find the median and the mean for these twelve figures.

### **Skills:**

- Students will understand how to find the areas of rectangles drawn on graph paper by counting the unit squares inside the rectangle.
- Students will be able to use inductive reasoning to generalize that counting the number of squares in each row and multiplying that number by the number of rows is more efficient and more accurate than counting each of the squares and they will derive the common simple formula for finding the area of a rectangle:

$$A = L \times W.$$

- Students will apply this simple formula  $A = L \times W$  to find the area of rectangles.

- Students will demonstrate their understanding of the problem-solving strategy of reducing a bigger, more complicated problem to a smaller, simpler problem by solving the Bowditch problem at the end of the lesson.
- Students will apply the problem-solving strategy of simplification by making quicker and more accurate calculations using techniques such as compensation, chaining, combination of compatible numbers, distributive law, regrouping.

### **Vocabulary:**

- Survey
- Population
- Density
- Perimeter
- Mean
- Area
- Median

### **Frameworks connections:**

#### **Mathematics**

##### **Strand 1: Number Sense**

- Standard 1.8:** Computation and Estimation (p. 42)
- Compute with whole numbers and fractions.
  - Use computation and estimation to solve problems.
  - Develop and analyze procedures for computing and estimating.

##### **Strand 2: Patterns, Relations and Functions**

- Standard 2.5:** Number Systems and Number Theory (p.61)
- Use operations involving whole numbers and fractions.
  - Apply number theory concepts.

##### **Strand 3: Geometry and Measurement (p.76)**

- Standard 3.4:** Measurement (p.76)
- Describe the meaning of perimeter, area, density.
  - Select the appropriate units and tools to measure the degree of accuracy required in a situation
  - Develop and apply formulas and procedures to solve measurement problems.

##### **Strand 4: Statistics and Probability**

- Standard 4.2:** Statistics (p.90)
- Collect, organize, and describe data systematically.
  - Construct, read, and interpret tables, charts, and graphs.

# Unit 7 Lesson Plans



## Lesson 1: Simple Areas

### Objectives:

- Students will find the area of rectangles
- with graph paper
- by applying the simple formula  $A = L \times W$

### Skills:

- Students will be able to use graph paper as a counting aid to find the area of rectangles.
- Students will understand how to use inductive reasoning to generalize that counting the number of squares in that row and multiplying the number by the number of rows is more efficient and more accurate than counting each of the squares individually.
- Students will derive the common simple formula for finding the area of a rectangle:  $A = L \times W$ .
- Students will apply the formula  $A = L \times W$  to find the area of rectangles.

### Vocabulary:

- Area
- Perimeter

### Materials:

- Graph paper
- Rulers
- Pencils
- Colored markers

### Procedure:

1. Distribute worksheet "Simple Areas". On the first page, have students find the area of each gridded rectangle by counting the squares inside the rectangle.
2. Ask the students, "What are the disadvantages of counting all of the squares individually? Is there an easier way to find the area of the rectangle without counting all individual squares?"
3. Students will derive the formula:  $A = L \times W$ .
4. On the second page of the worksheet, students will find the area of the rectangles by applying the new formula. They may check their answers by drawing the rectangles on graph paper and counting the squares.
5. Discuss perimeter and explain the distinction between perimeter and area. Ask students to find the perimeter of each rectangle.

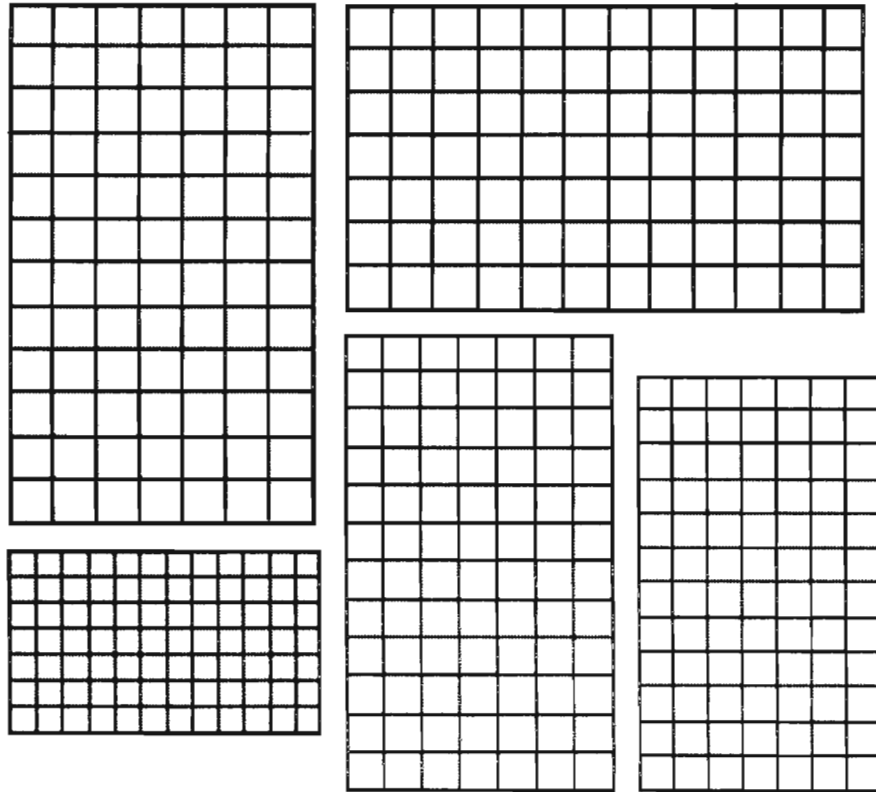
### Handout:





## SIMPLE AREAS

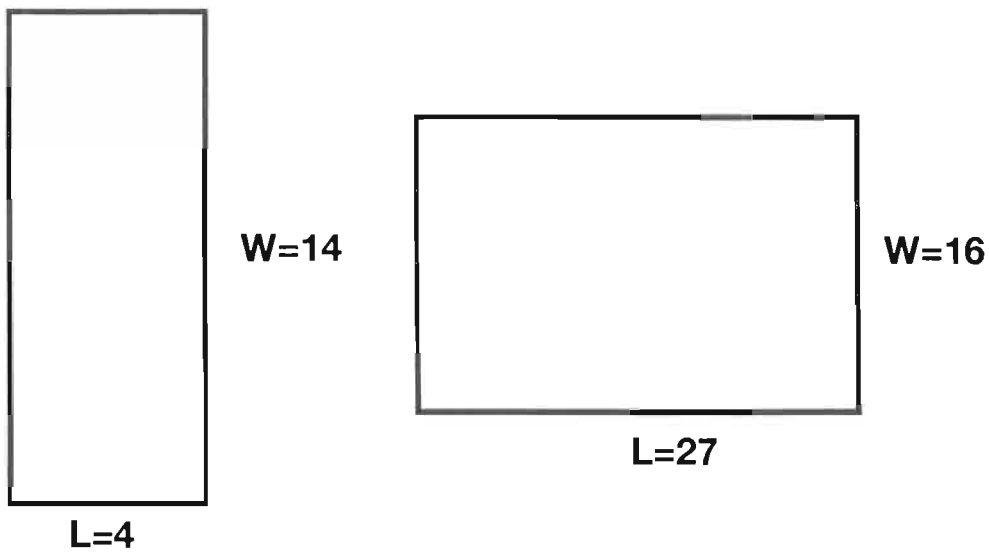
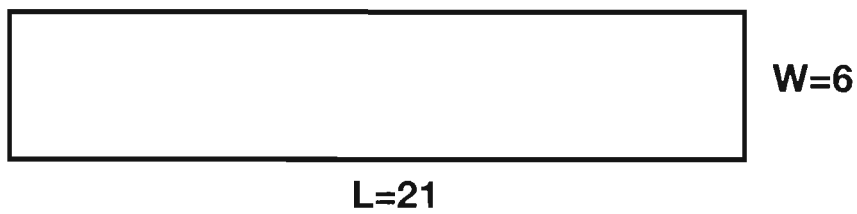
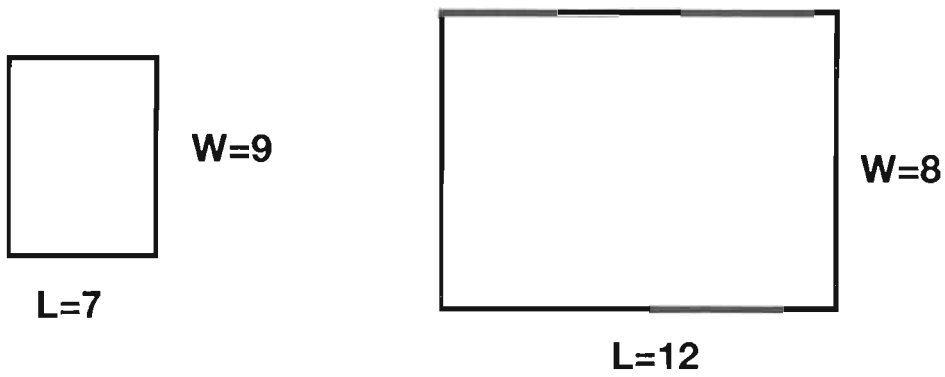
Look at the rectangles below. How could you find the area of each one? You could count the square units inside each rectangle, but that could be pretty boring if you had a really big rectangle and you might lose count. What would be an easier way to "count" the squares inside each figure? (Teachers: Have the students count the blocks in each figure).



You could count the number of squares in each row and multiply that number by the number of rows. Figure 1 has 7 squares in each row and 12 rows. The area would be  $7 \times 12 = 84$  square units. To find the area of any rectangle, we can use a simple formula:

$$A = L \times W.$$

This means area equals length times width. Use this formula to find the area of the rectangles on the next page.





## Lesson 2: Odd Areas

### Objectives:

- Students will find the area of odd shapes by breaking the area into rectangles and adding up the individual areas.
- Students will find the area of odd shapes by using graph paper.
- Students will demonstrate their understanding of the problem-solving strategy of breaking a bigger, more complicated problem into a smaller, simpler one by solving the Bowditch problem at the end of the lesson.

### Skills:

- Students will know how to use the problem-solving technique of simplification as they break down odd shapes into rectangles.
- Students will be able to use the formula  $A = L \times W$  to find the area of rectangular areas.
- Students will learn to use creativity in problem-solving by brainstorming with a team and comparing results with others.

### Vocabulary:

- Problem-solving strategy - simplification
- Area
- Survey

### Materials:

- Graph paper
- Rulers
- Pencils
- Colored markers

### Procedure:

1. Distribute worksheet, "Odd Areas".
2. Break students into groups of three or four. Encourage the students to brainstorm until they come up with the idea that the figures need to be broken down into rectangles, because they know how to find the area of rectangles. There are several ways to divide up each figure-creative differences are okay! Everyone can still come up with the right solution. Make sure they add up all the sub-areas for each figure and do not add any sub-areas more than once. Answer can be verified by drawing the figures on graph paper and counting the blocks.
3. Distribute worksheet "A Resourceful Young Man".

4. Proceed as with the activity in Step 2. Encourage students to think creatively about a solution to this problem, which involves breaking the problem down into simpler, more manageable parts. Ask them for what kind of shapes they could use to find the area. When they discover they can find the area of rectangles, ask them how this figure could be divided into rectangles and if they have enough information to solve the problem.

**Handouts:**

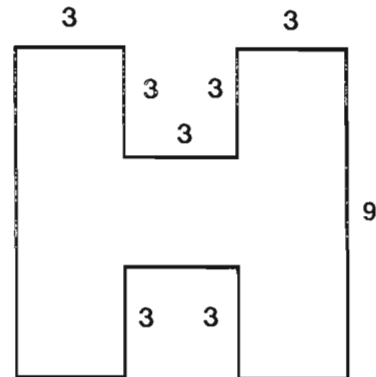
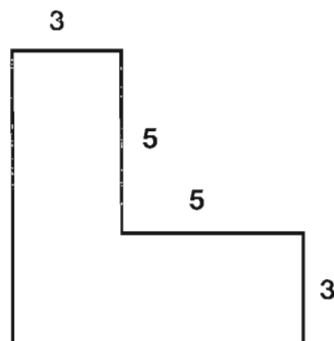
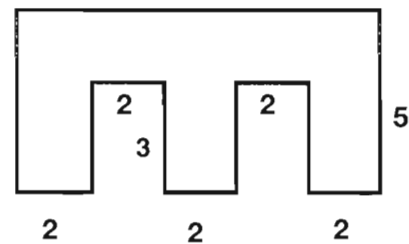
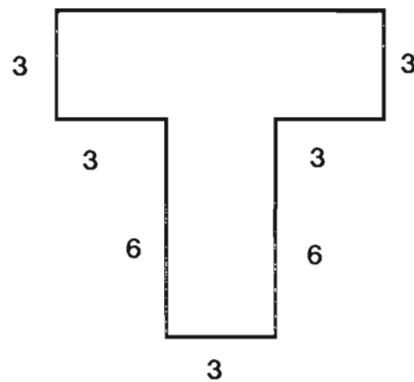
"Odd Areas"

"A Resourceful Young Man"

## Odd Areas

How would you find the area of the odd-shaped figures below? We have a formula for finding the area of a rectangle, but we don't have formulas for finding the area for figures shaped like a 'T', an 'L', an 'E' or an 'H'. If they were drawn out on graph paper, that would help, because you could count the squares. Graph paper is not always available, though. How could you solve it without graph paper?

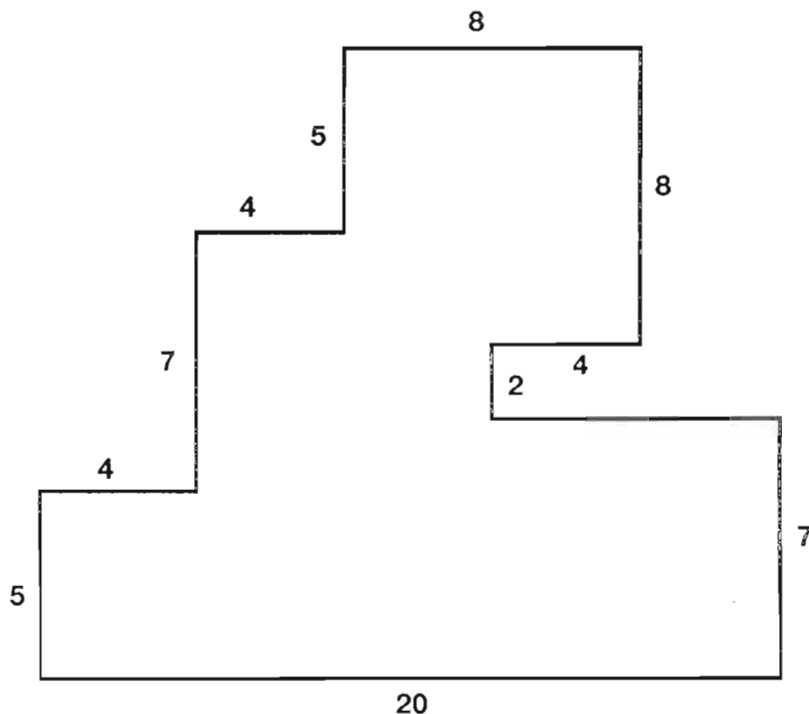
(Teachers: Encourage the students to 'brainstorm' until they come up with the idea that the figures need to be broken down into rectangles, because we know how to find the area of rectangles. There are several ways to divide up each figure---creative differences are okay! Everyone can still come up with the right solution. Make sure they add up all the sub-areas for each figure and don't add any sub-areas more than once. Answers can be verified by drawing the figures on graph paper and counting the blocks.)



## A Resourceful Young Man

As a teenager, Nathaniel Bowditch worked for a Mr. Hodges. During this time, Mr. Hodges had a dispute with a neighbor over an oddly-shaped parcel of land. Knowing what a clever mathematician his employee Bowditch was, Mr. Hodges called on Nat to survey (measure) his land and divide it equally between Hodges and his neighbor. Nat Bowditch thought about the problem for a while, made very careful measurements and was able to figure out the area and give Hodges and the neighbor each the same amount of land. The neighbor was upset, because he thought Hodges' portion was larger and that Bowditch had decided in Hodges' favor because he worked for him. So, they called in a professional surveyor. The surveyor's figuring gave Hodges an even bigger piece of the property! The neighbor should have listened to Nathaniel Bowditch!

Let's say the property in question looked something like this:

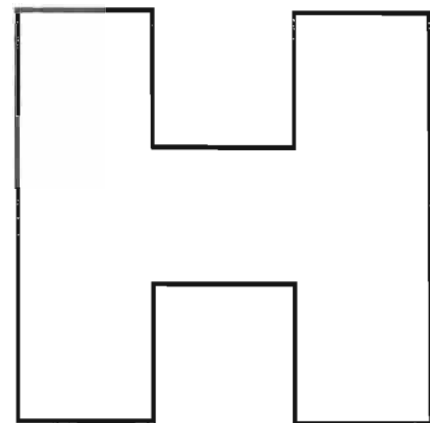
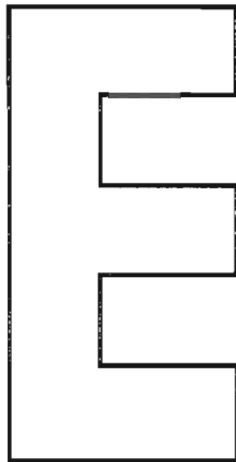
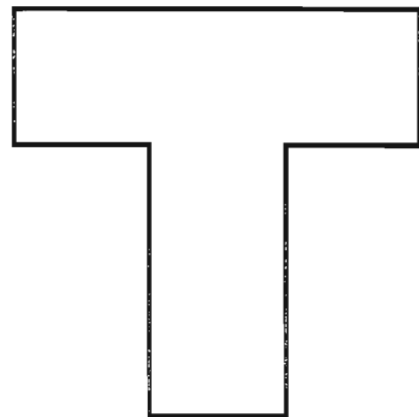
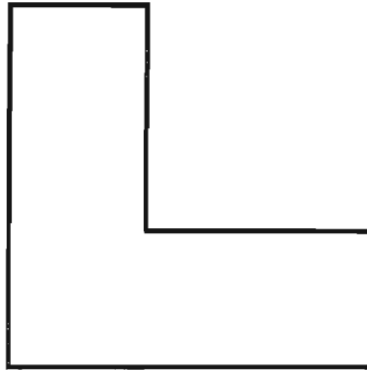


How did Bowditch do it?

Encourage students to think creatively about a solution to this problem, which involves breaking the problem down into simpler, more manageable, parts. Ask them what kind of shapes they could find the area of. When they discover they can find the area of rectangles, ask them how this figure could be divided into rectangles and if they have enough information to solve the problem. (They do.)



# ODD AREAS



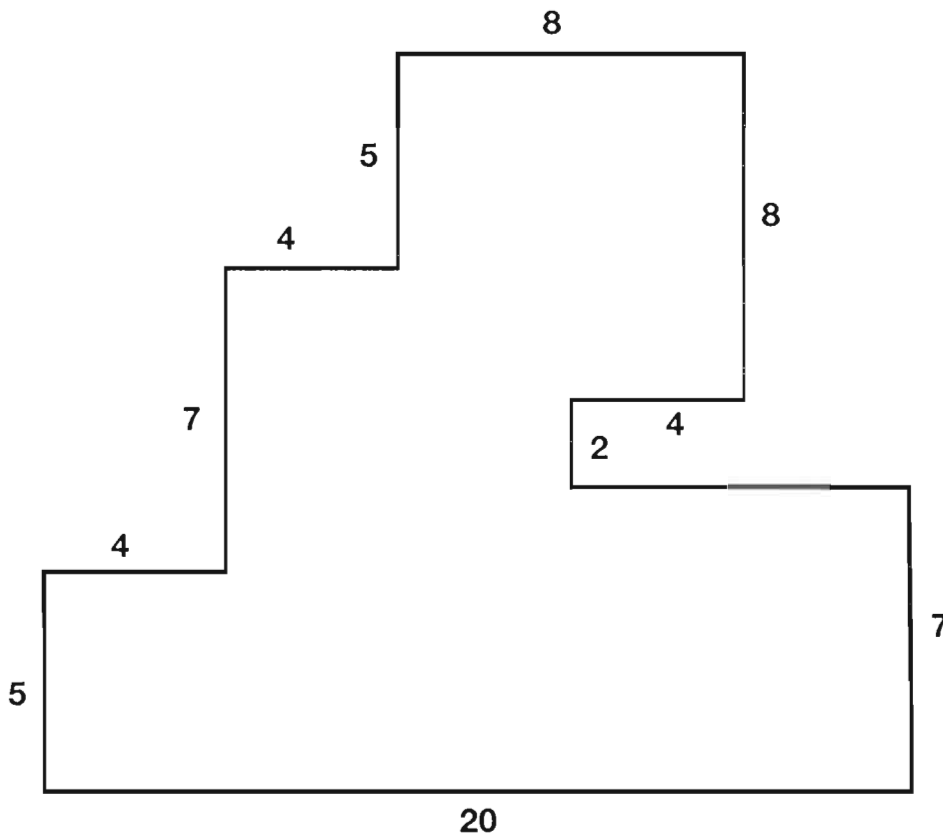
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## A RESOURCEFUL YOUNG MAN

As a teenager, Nathaniel Bowditch worked for a Mr. Hodges. During this time, Mr. Hodges had a dispute with a neighbor over an oddly-shaped parcel of land. Knowing what a clever mathematician his employee Bowditch was, Mr. Hodges called on Nat to survey (measure) his land and divide it equally between Hodges and his neighbor. Nat Bowditch thought about the problem for a while, made very careful measurements and was able to figure out the area and give Hodges and the neighbor each the same amount of land. The neighbor was upset, because he thought Hodges' portion was larger and that Bowditch had decided in Hodges' favor because he worked for him. So, they called in a professional surveyor. The surveyor's figuring gave Hodges an even bigger piece of the property! The neighbor should have listened to Nathaniel Bowditch!

Let's say the property in question looked something like this:



How did he do it?

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## Lesson 3: Population Density

### Objectives:

- Students compare the populations, areas and average house prices of twelve North Shore communities.
- Students will find the population density of each of the twelve communities by dividing the number of people in each town by the number of square miles in the town.
- Students will work with two types of averages-the mean and the median-as they find the mean house price and the median house price for these twelve communities.

### Skills:

- Students will get practice reading a table and find the information they need to solve a problem.
- Students will get practice rounding data, and arrange data from lowest to the highest values
- Students will be able to compare and contrast density data from twelve communities
- Students will know how to calculate simple statistics: mean, median, mode.

### Vocabulary:

- Mean
- Median
- Mode
- Population density

### Procedure:

1. Students will arrange the data in order from lowest to highest for population, area, and house prices.
2. Students will find the population density of each of the twelve communities and determine which community has the highest population density and which has the lowest population density.
3. Students will find the mean house price for the list by adding up all of the house prices on the list and dividing by 12 (the number of communities in the list). They will then decide which communities on the list have house prices closest to the mean house price for the area.



4. Students will find the median house price for the list by arranging the house list in order, adding the 6th and 7th items and dividing by 2. They will then decide which communities on the list have house prices closest to the median house price for the area.
5. Optional activity: Extend the concept of averages to the mode, the most frequently appearing number in a list of numbers.
6. You could use the ages of the students in the class as a database.

**Handouts:**

Twelve North Shore Communities

## Twelve North Shore Communities

Community	Population	Area (sq. mi.)	Average House Price
Beverly	33,855	15.14	\$210,600
Danvers	24,095	13.8	\$202,300
Gloucester	28,716	26.0	\$213,700
Lynn	81,245	13.5	\$159,900
Manchester-by-the-Sea	5,569	7.73	\$455,900
Marblehead	20,344	4.4	\$316,000
Middleton	6,534	14.4	\$220,000
Peabody	48,752	17.0	\$176,000
Rockport	7,776	7.0	\$260,000
Salem	37,735	8.18	\$151,000
Swampscott	14,006	3.05	\$256,360
Wenham	4,467	8.21	\$312,487

1. Which community listed above has the largest population? The smallest population?
2. Which community has the largest area? The smallest area?
3. Which community has the highest average house price? The lowest average house price?
4. Find the population density for each of the communities listed above. Population density is the number of people per square mile. To find the population density, divide the number of people by the number of square miles. (You may use a calculator for this activity.) Which community listed above has the highest population density (most people per square mile)? The lowest population density (fewest people per square mile)?
5. Round the population for each community listed above to the nearest thousand. Round the area for each community to the nearest whole number of square miles. On graph paper, draw an area that has the same number of blocks as your community has square miles. (For example,

Wenham would be made up of 8 blocks and Beverly would contain 15.) Now, draw as many dots in the area as your community has thousands of people. (Wenham would have 4 dots; Beverly would have 34.) Make your dots uniform---i.e., the same size. Do this for a neighboring community and compare.

6. List the communities and their house prices in order from lowest to highest house prices. Find the median house price on this list by choosing the middle two numbers in the list (here, the 6th and 7th), adding them up and dividing by 2. (If there are an odd number of items in a list, the median is simply the middle number once the list is in order.) The median is the middle number of an ordered list (just as a median on a highway is the middle strip) and is a type of average. There are just as many numbers above the median as there are below it.
7. Now, let's find a different type of average---the one you are probably most familiar with: the mean. To find the mean, add up all the numbers and divide by the number of entries, in this case, 12. Which community has house prices closest to the mean?

## Twelve North Shore Communities Answers

1. largest population-Lynn  
smallest population-Wenham
2. largest area-Gloucester  
smallest area-Swampscott
3. lowest-Salem  
highest-Manchester-by-the-Sea
4. Answers below:

Community	Population Density
Beverly	2236
Danvers	1746
Gloucester	1104
Lynn	6018 (highest)
Manchester-by-the Sea	720
Marblehead	4624
Middleton	454 (lowest)
Peabody	2868
Rockport	1111
Salem	4613
Swampscott	4592
Wenham	544

5. Teachers should check students' work in rounding off a particular community's population and blocking it off on graph paper.
6. median house price-\$216,850; Gloucester is closest.
7. mean house price-\$210,237.25; Beverly is closest.





## TWELVE NORTH SHORE COMMUNITIES

Community	Population	Area (sq. mi.)	Average House Price
Beverly	33,855	15.14	\$210,600
Danvers	24,095	13.8	\$202,300
Gloucester	28,716	26.0	\$213,700
Lynn	81,245	13.5	\$159,900
Manchester-by-the-Sea	5,569	7.73	\$455,900
Marblehead	20,344	4.4	\$316,000
Middleton	6,534	14.4	\$220,000
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Salem	37,735	8.18	\$151,000
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Wenham	4,467	8.21	\$312,487

1. Which community listed above has the largest population? The smallest population?
2. Which community has the largest area? The smallest area?
3. Which community has the highest average house price? The lowest average house price?
4. Find the population density for each of the communities listed above. Population density is the number of people per square mile. To find the population density, divide the number of people by the number of square miles. (You may use a calculator for this activity.) Which community listed above has the highest population density (most people per square mile)? The lowest population density (fewest people per square mile)?



5. Round the population for each community listed above to the nearest thousand. Round the area for each community to the nearest whole number of square miles. On graph paper, draw an area that has the same number of blocks as your community has square miles. (For example, Wenham would be made up of 8 blocks and Beverly would contain 15.) Now, draw as many dots in the area as your community has thousands of people. (Wenham would have 4 dots; Beverly would have 34.) Make your dots uniform---i.e., the same size. Do this for a neighboring community and compare.
6. List the communities and their house prices in order from lowest to highest house prices. Find the median house price on this list by choosing the middle two numbers in the list (here, the 6th and 7th), adding them up and dividing by 2. (If there are an odd number of items in a list, the median is simply the middle number once the list is in order.) The median is the middle number of an ordered list (just as a median on a highway is the middle strip) and is a type of average. There are just as many numbers above the median as there are below it.
7. Now, let's find a different type of average---the one you are probably most familiar with: the mean. To find the mean, add up all the numbers and divide by the number of entries, in this case, 12. Which community has house prices closest to the mean?

## UNIT 8:

# NAVIGATION



## OVERVIEW FOR TEACHERS

### Unit Outline

Part of the lore of Nathaniel Bowditch is the story of his remarkable navigation through fog and rock into Salem Harbor on December 25, 1803.

Nathaniel's ship, *The Putnam*, encountered severe fog off of Nantucket Island. At 4 PM on Christmas Day, the fog lifted only briefly, allowing Bowditch to sight Cape Ann. A glimpse of Baker's Island several days before and one sextant sighting of the sun were enough knowledge for Bowditch to successfully navigate the treacherous Middle Ground, and finally home to Salem, still shrouded by its infamous "pea soup" fog.

### Introduction

Nathaniel Bowditch and navigation are synonymous. The renowned author of the still-published *New American Practical Navigator* is also synonymous with self-education. When one reviews Nathaniel's life accomplishments in navigation, astronomy, and mathematics, it is humbling to recount his early years indentured to the chandlery of Ropes and Hodges. Nat's encounters with Dr. Bentley and Dr. Prince, members of the Salem Philosophical Society, and their invitation to use the Society's outstanding scientific library, were a direct result of this indentureship. Nat had opportunities to experience the greatest minds and writings available at that time; opportunities not necessarily afforded Harvard men. The indentureship also placed Nat with his

Every seaman knew the necessity for accurate navigation. For a mind like Bowditch, the study of navigation was the perfect marriage between his favorite subjects, mathematics and astronomy. Thus, the indentured twelve-year old, confined to a busy ship chandlery in the worldly port of Salem, fed his inquisitive mind and mathematical genius with knowledge from an outstanding library, daily encounters with the nautical world, and a natural perseverance for self-betterment.

The following unit on Navigation highlights several topics important to successful navigation. The lessons introduce contour mapping, plotting a course, using triangulation, and using ratio and proportion to determine measurement. Students will also simulate the use of chains for determining water depth. Finally, students will create an essential instrument to navigators, the quadrant. They will determine their home latitude using their measurements of *Polaris*, the North Star.

**Objectives:**

- Students will be able to use standard and non-standard units.
- Students will be able to use a contour map to plot a course.
- Students will be able to find location by triangulation.
- Students will use ratio and proportion to determine measurement.
- Students will construct a quadrant and estimate their home latitude.

**Skills:**

- Students will understand how to use ratio/proportion to convert depth readings •
- Students will be able to use geometry to triangulate their location and determine their latitude.

**Vocabulary**

- Depth
- Fathom
- Triangulation
- Quadrant

## **Frameworks Connections:**

### **History and Social Science:**

#### **Strand 2: Geography**

##### **Standard 7: Physical Space of the Earth**

- Student will be able to visualize and map oceans and continents (p. 94).

##### **Standard 8: Places and Regions of the World**

- Identify and explain features (p. 96).

### **English Language Arts:**

#### **Composition**

- Write a composition with clear focus (19.6, p. 48)

### **Mathematics:**

#### **Strand 1: Number Sense**

##### **Standard 1.8**

- Estimate to solve problems (p. 42).
- Apply ratio and proportion (p. 40).

#### **Strand 3: Geometry**

##### **Standard 3.4**

- Select appropriate unit (p. 76).

# Unit 8 Lesson Plans



## Lesson 1: Non-standard Measurements

### Objectives:

- Students will use standard and non-standard units.

### Skills:

- Students will be able to use ratio/proportion to convert depth readings

### Vocabulary:

- mean
- median
- mode
- fathoms
- meters
- non-standard measurement
- standard measurement

### Materials:

- Adding machine tape

### Procedure:

#### Activity:

1. Discuss with the students "non standard measurement" / history connection.
2. Each student should measure 5 "hands" across a piece of adding tape.
3. Each student should measure the same length in the classroom ( a window, a desk, etc).
4. Represent that length in "hands". Write the values on the board.
5. Graph the data (line plot, bar graph).
6. Extension: Find the mean, median, mode of the data.

#### Writing Activity:

1. What are the advantages/disadvantages of using a non-standard measurement like a "hand"?
2. Why is it important to know what standard of measurement is used on a chart?
3. Show how to convert:
  - a. 20 fathoms into feet;
  - b. 13 feet into fathoms,
  - c. 100 meters into fathoms



## Lesson 2: Contour Mapping

[\*Reference/ *Charting Our Course*, "Contour Mapping Activity" p 7]

### Objectives:

- Students will use a contour map to plot a course.
- Students will find location by triangulation.
- Students will use ratio and proportion to determine measurement

### Skills:

- Students will be able to use ratio/proportion to convert depth readings

### Vocabulary

- Depth
- Fathom
- Triangulation

### Materials:

- map of Salem Sound/Salem Harbor or map of Gulf of Maine
- foam board or layers of cardboard
- scissors
- glue

### Procedures:

1. Before class: For each group, make 4 photocopies of the chosen map: one of each in white, blue, yellow, green for each group
2. See Activity #2 sheet
3. Field trip possibility: Peabody Essex Museum/ Bowditch Room
4. Sources for the reading of *The Putnam's* return in a snowstorm:
  - i. Berry, Robert. *Yankee Stargazer: The Life of Nathaniel Bowditch*
  - ii. Latham, Jean. *Carry on Mr. Bowditch.*
  - iii. Rink, Paul. *To Steer by the Stars: The Story of Nathaniel Bowditch.*
  - iv. Stanford, Alfred. *Navigator: The Story of Nathaniel Bowditch.*
5. Source for the contour mapping activity:
  - i. *Charting Our Course: The Massachusetts Coast at an Environmental Crossroads. Massachusetts Coastal Zone Management/ Massachusetts Marine Educators*





## ACTIVITY #2 CONTOUR MAPPING

### Materials for each group:

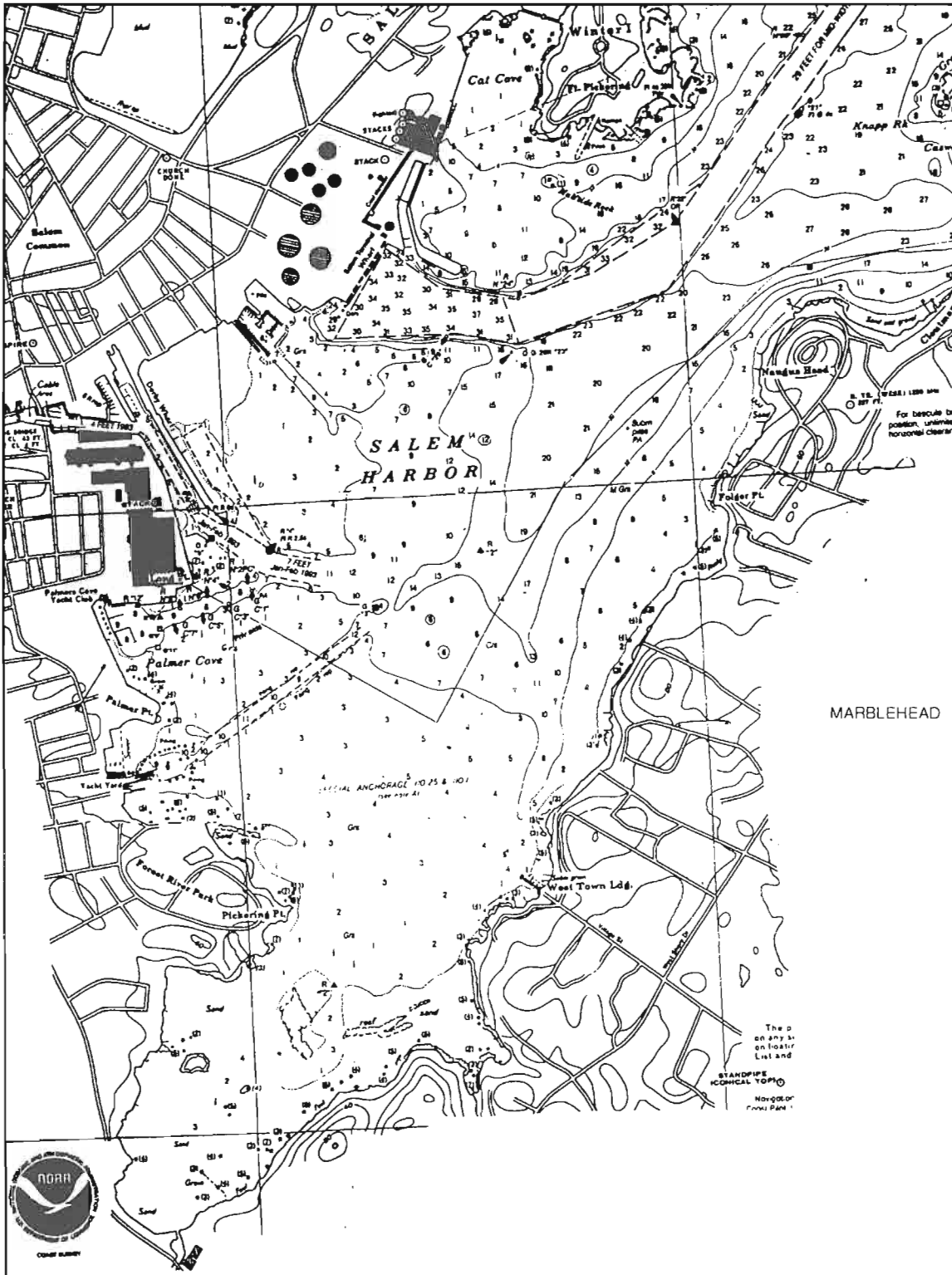
- 4 copies of Salem Sound, each on a different color paper
- scissors
- glue
- foam board (or cardboard)
- A nautical chart of Salem Sound
- X-acto knife

### Procedure:

1. Study the white copy. This is your base map. Glue it to the foam board. Cut the whole map out carefully.
2. Use the blue copy to cut along all the 30 feet contour lines. Throw away the sections representing areas deeper than 30 feet. Glue the blue pieces onto the foam board. Cut the blue pieces out and glue onto the matching part of the white base map.
3. Repeat the procedure using the green paper and the 20 feet contour. Throw away areas deeper than 20 feet. Glue the green to the foam board, cut out carefully and glue onto the blue.
4. Repeat a final time with the yellow paper. Cut along the edge of the land and the 10 foot contour line. Throw away anything deeper than 10 feet. Glue this carefully in place.
5. On the land, note any navigational landmarks such as the stacks at the power plant or the lighthouse at Fort Pickering.
6. Listen to a selection that describes the arrival of *The Putnam* on Christmas Day in the fog.
7. Write a descriptive paragraph about the sail into Salem Harbor as seen by a member of the crew of *The Putnam*. Use adjectives and adverbs to describe the experience.



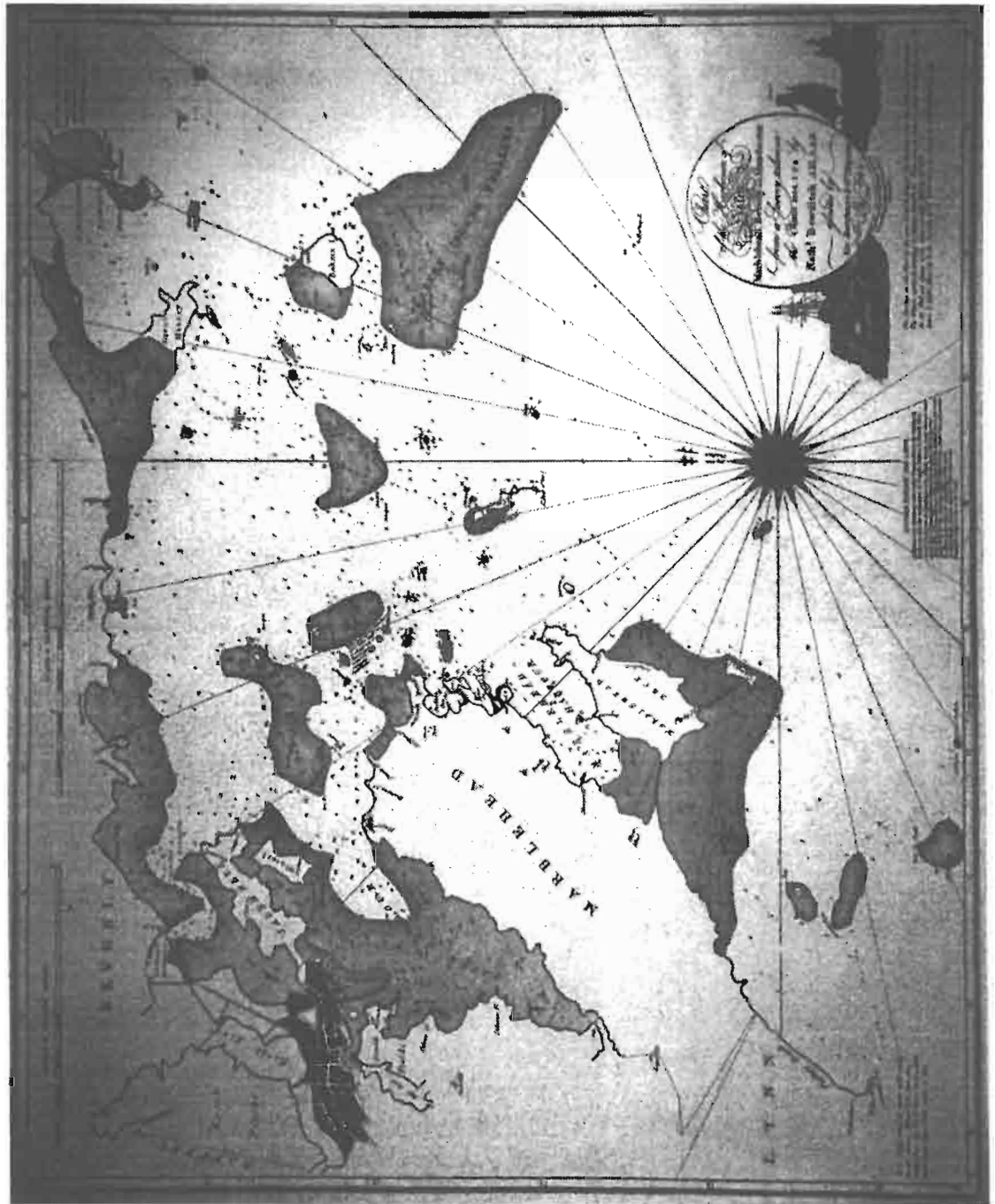
UNIT  
8  
STUDENT  
HANDOUT



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# ORIGINAL BOWDITCH MAP



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*Courtesy of and copyright of the Peabody Essex Museum, Salem, MA*





## Lesson 3: Triangulation

### Objectives:

- Students will use geometry to triangulate their location and determine their latitude.

### Skills:

- Geometry of angles

### Vocabulary:

- Triangulation

### Materials:

- a model of a large compass to put on the floor
- a navigational chart of Salem Sound (Chart 13276 from NOAA is recommended)
- rulers
- pencils
- activity worksheets

### Procedure:

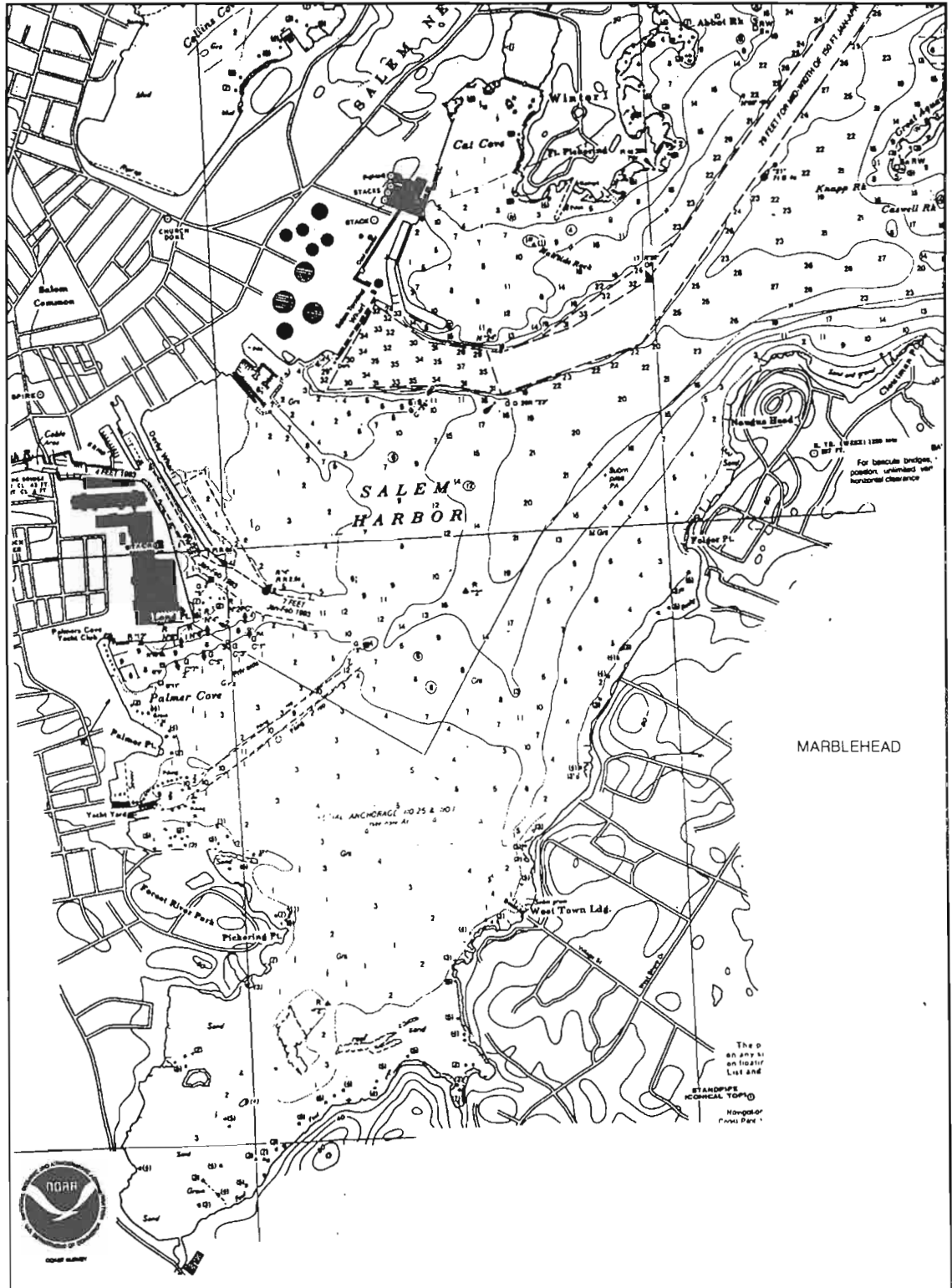
Background: Triangulation is a navigational method of locating your position on a nautical chart if you are able to sight two other landmarks. To find your location, you must first use your ship's compass to sight two landmarks (lighthouses, marked buoys, an island, etc.) These readings are then transferred to the compass rose on the navigational chart. Parallel lines are drawn and your location is at the intersection of the lines. A good introductory activity is *The Voyage of the Mimi: Maps and Navigation* (Sunburst Communications, Inc, 1985).

1. Introduce students to the bearings on a compass with the group activity "Getting Your Bearings."
2. Students practice using the compass rose and parallel lines on a map: "Chart Activity #1".
3. Students use the map of Salem Sound to locate position by triangulation: "Chart Activity #2".
4. Extension: Have students list the navigational hazards in Salem Sound.
5. Compare the location of the Salem and Marblehead Channels to these hazards.



UNIT  
8  
STUDENT  
HANDOUT

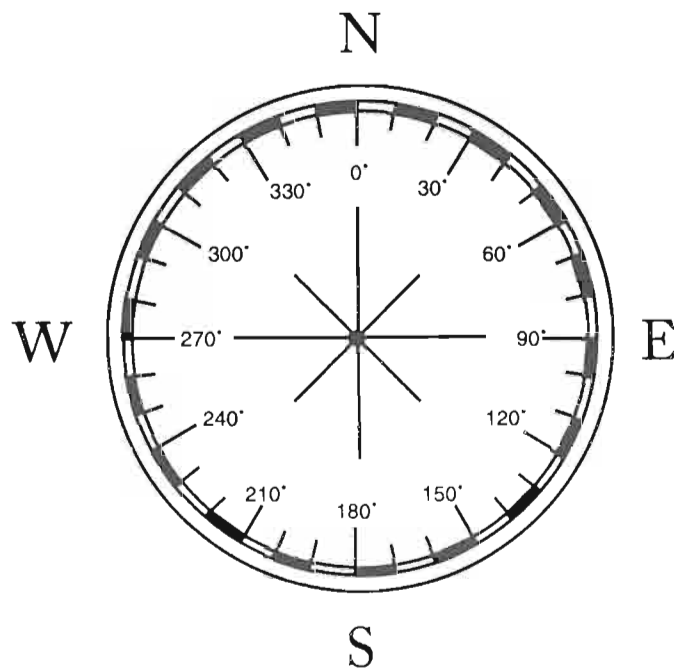
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# GETTING YOUR BEARINGS

**Background:** A compass is a large circle marked off in degrees. There are 360 degrees in a circle and there are 360 degrees on a compass' face. The compass rose on a navigational chart looks like the face of a compass and shows where north is. The major directions are north (N), south (S), east (E), and west (W). These can be broken down into smaller units like northeast (NE) or north northeast (NNE). Look at the diagram below of a compass rose.

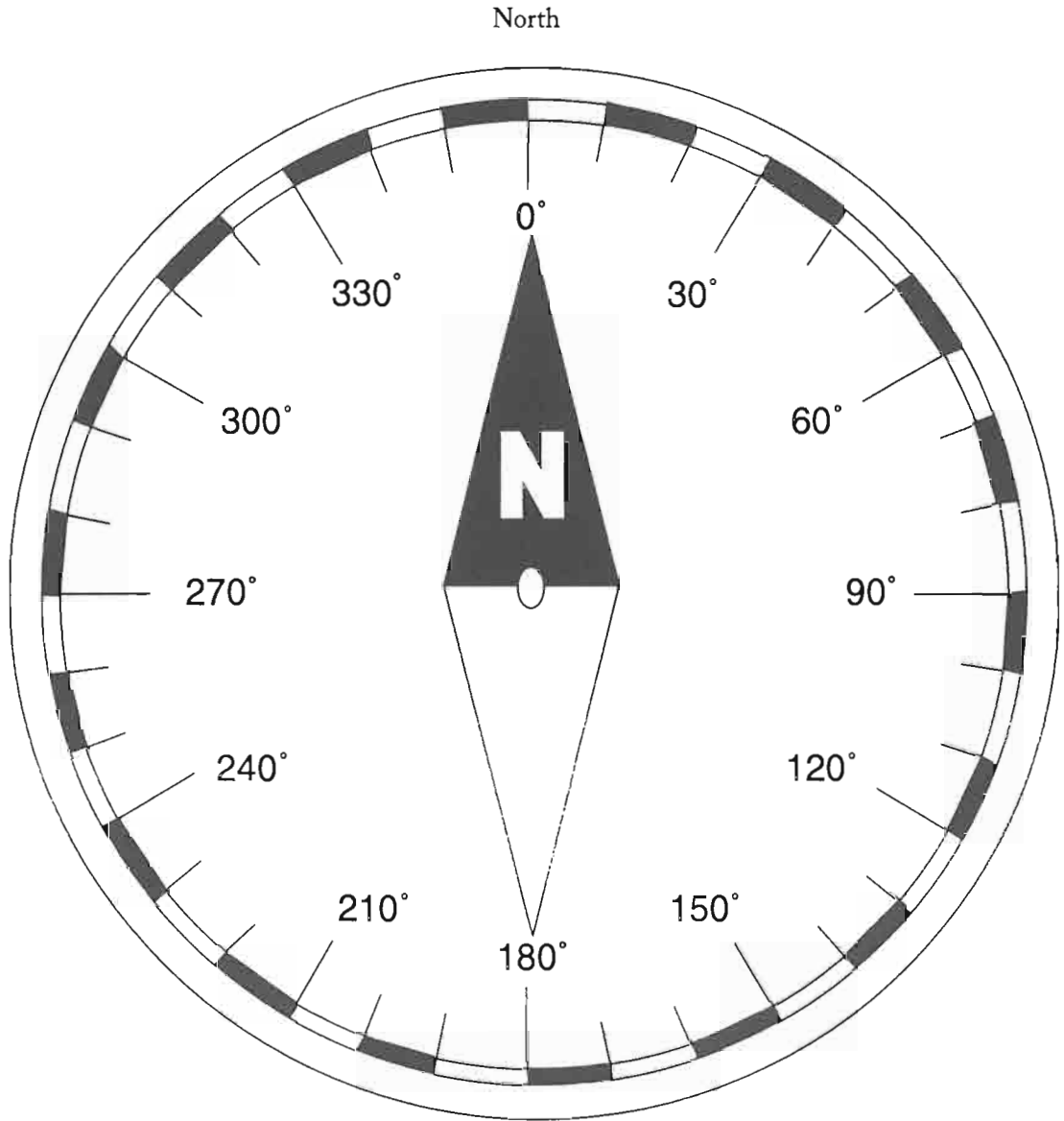


**Procedure:** In a large area, make a circle around the compass rose on the floor. Choose small objects to put around the circumference of the circle ( A book, a wastebasket, a plant, etc) One student should stand in the middle of the compass rose holding one end of a long piece of string. Students should practice sighting along the tautly stretched string.

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# GETTING YOUR BEARINGS WITH COMPASS



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## CHART ACTIVITY #1

Use what you have learned about sighting on the compass and a chart of Salem Sound to answer the following questions.

1. If you are on Bakers Island, in what direction will you look to see Bowditch Ledge?
2. You are at the summer camp on Cat (Children's) Island. Your boat requires 10 feet of water at all times. Would you motor north to Bakers Island? Explain.
3. Your sailboat requires 4 feet of water at all times. You are located at Green can #3 off Chappel Ledge to the northwest of Cat Island. Describe the safest route to Mackerel Cove in Beverly.
4. Many local Salem sailors avoid the South Channel when sailing to Marblehead at low tide. Is this "local legend" or based on fact? Back up your opinion with details.



## CHART ACTIVITY #2

Like Bowditch on *The Putnam*, you have sailed between Bakers Island and Great Misery Island into Salem Sound. You need to locate your position on the navigational chart before you head for Salem Harbor.

1. You notice the large stacks at the Salem Power Plant and see them on your chart. Your compass reading is  $260^{\circ}$ .
2. You also note a buoy, Red Nun #6. Its compass heading is  $205^{\circ}$ .
3. You will need a clear ruler and a pencil. On the chart's compass rose, place the ruler on the exact center and work out to the reading for the stacks at the power plant,  $260^{\circ}$ . Draw a faint line. Now carefully slide your ruler towards the stacks on the chart trying to keep the ruler even or parallel to the original heading. When you reach the stacks, draw a long line into the Sound that will be parallel to the line through  $260^{\circ}$ . You are located somewhere on that line.
4. The bearing to the Red Nun is  $205^{\circ}$ . Repeat the process in #3 with this new bearing. Your ship is located where the two lines intersect.
5. Using compass headings, describe the course you would order to sail safely into Salem Harbor at mean low tide.





## Lesson 4: Depth Measurements

### Objectives:

- Students will simulate depth soundings by chains

### Skills:

- Students will learn to simulate sea-floor mapping

### Vocabulary:

- Chains
- Depth Sounders
- Salem Sound

### Materials:

- Several large fish tanks (no water is needed inside the tanks)
- Opaque paper or black paint
- Sand
- Rocks, balls
- Grid sheet
- Long knitting needles
- Pencils

### Procedure:

1. Background: Today mariners use depth sounders to determine how deep the water is around them. In Bowditch's time, chains were thrown overboard and the water height was marked to indicate depth. Look at a navigational chart of Salem Sound and note the depth of water at mean low tide.
2. An exercise to simulate this method can be done with a large fish tank. Cover the outside of the tank with opaque paper or paint it black.
3. Cover the bottom with sand and then place objects on the sand such as tall rocks or balls. These simulate underwater hazards.
4. Cover the top of the tank with a thin, dark covering such as a black gauze curtain.
5. Give the students a grid sheet marked off in squares; the area of the grid should equal the area of the floor of the fish tank.
6. Give one student a long knitting needle and have them put the needle through the covering until it strikes a solid.
7. That depth is marked and noted on the grid.

8. This process is repeated over the entire surface area of the tank ; the students then speculate about the surface of the floor.
9. The knitting needle represents the chains the mariners used to find depth.
10. This activity is best done with several tanks. Water is not needed in the tank.

**Alternative materials and procedure:**

1. Gather large empty coffee cans with plastic covers - enough for every two students.
2. Create a grid on the top of each plastic cover.
3. Fill the cans in the same way that you would fill the fish tanks.
4. Using a sharp wooden skewer, pierce the plastic tops at each point on which the vertical and horizontal lines meet. Push skewer down carefully, to determine the exact place where the skewer meets an obstacle on the bottom of the coffee can.
5. Mark one of the axes on the outside of the can. Then graph the contour of the "ocean floor" on graph paper. Only after completing this task are students allowed to open the can and compare their graph to the actual contours.

## UNIT 9

# MATHEMATICAL SKILLS AND HABITS



## OVERVIEW FOR TEACHERS

### Unit Outline

#### Introduction:

*In addition to the text, the explanations and the diagrams, the thousands upon thousands of extremely complicated computations for the tables drained all his strength and time. This was long before any such thing as a mechanical calculator, so every formula had to be laboriously written on the slate. Since he [Nathaniel Bowditch] was determined that above all, his book should be accurate, he worked each set of figures three times. (To Steer by the Stars, p. 166)*

Nathaniel Bowditch was a natural mathematician—he found joy in counting, measuring, estimating, calculating, thinking logically, and looking for patterns. Nat also paid attention to details. Early in his life, he developed good habits - counting carefully, measuring accurately, and calculating over and over again to insure accurate answers. His good habits saved many lives, as well. The accuracy of Bowditch's *New American Practical Navigator* allowed ships to navigate with confidence; the mathematical tables that their lives

A few mistakes can lead to many other errors. Bowditch examined and recalculated the commonly used navigation tables of the early 1800's and discovered over 8000 mathematical errors! Scientists and mathematicians know that simply one or two errors in computation or measurement can lead to drastic miscalculations and conclusions. In the following unit, students will discover how one miscalculation in a checkbook entry can cause all the other entries to be wrong until the error is corrected. Students will apply their understanding of miscalculations to potential navigation errors. The mismeasurement of an angle by only a few degrees can lead sailors hundreds of miles off course, landing them in another country, perhaps even another continent!

Bowditch's mathematical skills made him invaluable as a supercargo on a ship:

*Soon his head was filled with numbers, translating everything about the cargo into columns of figures, into addition, subtraction, multiplication, and division. The numbers flew through his head in a swirling tide of calculations in which he computed weights, values, units of money, money exchange of foreign countries, duties and many other things.*

(*To Steer by the Stars*, p. 23)

Students will practice their own computational skills as they convert percents to decimals or fractions. They will solve problems involving weight conversions, requiring skill in determining proportions. Finally, students will calculate duties, taxes and rates of foreign exchange, skills essential to the success of any Salem merchant ship.

### **Objectives:**

- Students will investigate and discuss jobs that require math skills.
- Students will examine and correct a budget problem and a checkbook problem.
- Students will measure and compare distances between lines when the angle is varied.

### **Skills:**

- Students will be able to practice their skills in counting, measuring, estimating, calculating, thinking logically, looking for patterns.
- Students will know how to practice good mathematical habits in solving the above problems as they:
  1. Pay attention to details

2. count, measure, calculate carefully, and
3. check their work.

- Students will be able to use a ruler and a protractor to construct and measure angles.
- Students will know how to use proportions to solve problems.
- Students will understand how to use deductive reasoning as they apply general rules to specific examples.
- Students will be able to convert percents to decimals and fractions in order to compute duties (taxes).

**Vocabulary:**

- |            |               |              |
|------------|---------------|--------------|
| • surveyor | • duty        | • supercargo |
| • percent  | • bookkeeper  | • accuracy   |
| • actuary  | • approximate |              |

**Frameworks connections:**

**Mathematics:**

**Strand 1: Number and Number Relationships**

**Standard 1.6 (p. 40)**

- Apply ratios, proportions, and percents.
- Represent and use equivalent forms of numbers, including fractions, decimals and percents.

**Strand 2: Computation and Estimation**

**Standard 1.3 (p.42)**

- Compute with whole numbers, fractions, decimals.
- Develop, analyze, and explain procedures for computing, estimating, and proportions to solve problems.

**Strand 3 : Measurement**

**Standard 3.4 (p. 76)**

- Select appropriate units and tools to measure the degree of accuracy required in a particular situation.
- Describe the concept of weights and other derived and indirect measurements.



**Strand 4 : Statistics****Standard 4.2** (p. 90)

- Collect, organize, and describe data systematically.
- Construct, read and interpret tables, charts, and graphs.

## Unit 9 Lesson Plans



### Lesson 1: The Importance of Accuracy

#### Objectives:

- Students will name jobs and careers that require mathematical skills (counting, measuring, estimating, calculating, thinking logically, looking for patterns).
- Students will practice good mathematical habits:
  - Pay attention to details.
  - Count, measure, calculate carefully.
  - Check their work.
- Students will discover that one or two mistakes in calculation or measurement can lead to many more, and that a small error in measurement can lead to huge miscalculations.

#### Skills:

- Students will be able to practice:
  - counting
  - measuring
  - estimating
  - calculating
  - thinking logically
  - looking for patterns
- Students will learn how to practice:
  - paying attention to details
  - counting, measuring, calculating carefully
  - checking their work
  - constructing and measuring angles and angular distances
- Students will understand how to use proportions and deductive reasoning.

#### Vocabulary:

- |            |              |              |
|------------|--------------|--------------|
| • surveyor | • bookkeeper | • supercargo |
| • actuary  | • accuracy   |              |

**Materials:**

- small plastic protractors, rulers, pencils
- big wooden protractor for chalkboard
- chalk, yardstick, masking tape
- world (political) maps
- Optional: rolling tape measure, masking tape

**Procedure:**

1. Read the story-problem about the trip to the mall, compare the results of the problem done correctly with the results of the problem done with several miscalculations.
2. Track entries on a page from a mock checkbook. On the first entry, ask the students to add when they should subtract, noting that not only is the first answer incorrect, but the rest of the answers are wrong as well.
3. Construct an angle of  $9^\circ$  with sides of 3 inches. Students will measure the distance between the sides of the angles at the tips of the rays ( $\frac{1}{2}$  inch). Extend the sides of the angles to 6 inches and measure the distance at the tips of the rays (1 inch).
4. A student should go to the chalkboard and use a large wooden protractor to construct an angle of  $9^\circ$  with sides of 18 inches. Measure the distance between the sides (3 inches). Is this the expected answer? (Yes.) What if the rays are extended to 24 inches? Discover the general rule that applies to this problem through deductive reasoning.
5. Take the students to a large area such as the gym, cafeteria or schoolyard. Two students should start from the same spot (marked by an 'X' in masking tape), but vary their course by 5 degrees. Walk 20 feet in a straight line. Measure the distance between them when they stop (kinesthetic learning).
6. Using a series of proportions, students can determine the distance between the rays after 12 inches or 1 foot (2 inches), after 60 feet (10 feet), after 5280 feet or 1 mile (880 feet), after 3000 miles (500 miles).
7. Problem: A ship from Salem, Massachusetts plotted a course toward Spain. The navigator made a mistake and calculated a course  $9^\circ$  SE of the correct bearing. Where will the ship land?

**Handouts:**

- Worksheet "Accuracy Is Important" (2pp.)

## Accuracy is Important Answers

Nathaniel Bowditch (1773-1838) had many jobs in his lifetime: surveyor, book-keeper, supercargo, actuary. These jobs require good math skills, such as counting, measuring, estimating, calculating, thinking logically, looking for patterns. Can you think of other careers that require math skills? What really made him a good mathematician, however, were some of his habits. He paid attention to details, he was very careful when he counted, measured or calculated and he checked his work! Bowditch knew accuracy was important.

In writing a book on celestial navigation, Nat worked carefully---sailors' lives depended on his careful calculations!

*In addition to the text, the explanations and the diagrams, the thousands upon thousands of extremely complicated computations for the tables drained all his strength and time. This was long before any such thing as a mechanical calculator, so every formula had to be made laboriously on the slate. Since he was determined that above all, his book should be accurate, he worked each set of figures three times. (To Steer by the Stars, p. 166)*

As a young man, Bowditch had studied the navigational tables that sailors used. He spotted some errors and corrected them. By the time he had finished the book of tables, he had found over 8000 errors! How could such a thing happen? A few mistakes can lead to many other mistakes, in this case, thousands of mistakes. In fact, just one or two errors in calculation can lead to many more errors. We'll see how that is so.

**Example 1:** Let's say you and your friends were going to spend an afternoon at the mall. You have \$40 to spend and you figure you'll be able to buy some CD's, have pizza and a soda, then see a movie. First, you purchase 3 CD's on sale for \$9.99 and miscalculate that the total will be \$27 instead of? (\$29.97) You subtract 27 from 40 and get \$13 left, but how much do you really have? (\$10.03) your share of the pizza and soda comes to \$6, which you subtract from the \$13 you think you have, and you miscalculate again, and now think you have \$9 left---just enough for the movie. How much do you really have? (\$4.03) Guess who's not going to the movies?

	Your Answer	Right Answer
start with	\$40.00	\$40.00
3 CD's @ \$9.99	-\$27.00	\$29.97
how much left	\$13.00	\$10.03
pizza and soda	- 6.00	- 6.00
how much left	\$ 9.00	\$ 4.03

**Example 2:** Many people have a checking account. Deposits (or credits) add money to the account. When a person writes a check (or debit) it subtracts money from the account. Here is an example where a person made a computational error just once, but it affected her entries (answers) for the next eight transactions. (Teachers: Have students start out with a balance of \$1000.00. Then have them 'write' out a check for \$240.00 and add it their balance instead of subtracting it. As a class, enter several more transactions, and have them add deposits and deduct check amounts correctly. Have them note their balance is still 'off', all because of one mistake in the beginning.)

[illegible]

You've seen how a little computation error or carelessness can get you into trouble. One measurement that is just a little bit off can also lead to a BIG mistake!

**Example 3:** Teacher: Using a big wooden protractor, draw on the chalk-board a  $30^\circ$  and a  $32^\circ$  angle from the same vertex. Make the rays of the angle 4" long. Have a student come up to the board and measure the distance between the tips of the angles. ( $1/4$ ") Write that result on the board. Now extend the rays to 12" (= 1 ft.) each, and have another student measure that difference. ( $3/4$ ") Write that result on the board. What if we make the rays 4' long? (Difference is now 3" long.) 16' long? (off by 12"=1') Now ask students what would happen if the sides of the angles were extended for a mile. Use a simple proportion to figure out how far off you'd be by then. (330') How far off would you be after 3200 miles? Encourage them to use simplification methods to do this Calculation. (200 miles! That could put you in another country or even on another continent!)

## Avoiding Errors

If you really want to excel in math, you must practice the habits which Nathaniel Bowditch practiced. You must:

- pay attention to details
- count, measure, and calculate carefully
- check your work

There are ways to make your math life easier.





## ACCURACY IS IMPORTANT

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	Your Answer	Right Answer
start with	\$40.00	\$40.00
3 CD's @ \$9.99	-\$27.00	
how much left	\$13.00	
pizza and soda	- 6.00	
how much left	\$ 9.00	



**Example 2:** Many people have a checking account. Deposits (or credits) add money to the account. When a person writes a check (or debit) it subtracts money from the account. Here is an example where a person made a computational error just once, but it affected her entries (answers) for the next eight transactions. (Teachers: Have students start out with a balance of \$1000.00. Then have them 'write' out a check for \$240.00 and add it their balance instead of subtracting it. As a class, enter several more transactions, and have them add deposits and deduct check amounts correctly. Have them note their balance is still 'off', all because of one mistake in the beginning.)

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- count, measure, and calculate carefully
- check your work

There are ways to make your math life easier.





## Lesson 2: Paying Duty

### Objectives:

- Students will calculate the duty, or tax, on the value of goods.

### Skills:

- Students will be able to read and use information from tables to solve problems.
- Students will know how to change percents to equivalent decimals.
- Students will understand how to change percents to equivalent fractions.
- Students will learn how to find the percent of a quantity by multiplying the quantity by a fractional or decimal equivalent of the percent.

### Vocabulary:

- duty
- percent

### Materials:

### Procedure:

1. Distribute worksheet "Paying Duty".
2. Show students how to convert percents to decimals. (see worksheet.)
3. Show students how to convert percents to fractions. (see work sheet.)
4. Present the challenge problem at the end of worksheet. Determine the various duties on ten items. Add the duties to find the total duty for the cargo. This activity can be worked in small groups of three or four students.

### Handout:

Worksheet "Paying Duty" (2pp.)

## Paying Duty

If a merchant ship returned safely to Salem after its voyages to far ports, it would be laden with exotic goods from all over the world. Wine, citrus and dried fruits from Spain, cocoa and ivory from Africa, cottons from India, coffee from Arabia, cinnamon, cloves and black pepper from the East Indies, porcelain, tea and silk from China filled Salem warehouses. This meant big profits for the ship owner, but first, a duty had to be paid. A duty is a type of tax paid on goods brought into a port. The collector for the port would assess (find the value of) the goods, then he would collect a certain percent of the value as duty.

When working with percents, you need to change the percent to an equivalent decimal or fraction. Converting a percent to a decimal is very simple: you simply drop the % sign and move the decimal point two places to the left. (If you do not see a decimal point, it is assumed to be at the end of the number.) For single digit numbers, write a '0' before the number before you drop the % sign and move the decimal point two places to the left. Here are some examples:

$$35\% = .35$$

$$4\% = 04\% = .04$$

$$125\% = 1.25$$

$$37.5\% = .375$$

Percents can easily be changed to a fraction when you know that percent mean "out of a hundred". For example,  $17\% = 17/100$ .  $4\% = 4/100 = 1/25$ . Remember always to simplify your work by reducing fractions to lowest terms.

Use the table below to figure out the problems on your worksheet. Here are a couple of examples:

Example 1: Find the duty on \$2500 worth of silk.

Solution: From the chart, the duty on silk is 20%.  $20\% = 20/100 = 1/5$ .

$1/5$  of \$2500 = \$500. ("of" with fractions mean "multiply")

Example 2: Find the duty on iron valued at \$600.

Solution: From the chart, the duty on iron is 5%.  $5\% = .05$ .

$.05 \times \$600 = \$30$ .

5%	10%	12%	20%
iron	porcelain	cinnamon	silk
zinc	salt	cloves	ivory
lemons	coffee	nutmeg	jade
limes	tea	ginger	gold dust
oranges	cocoa	black pepper	
dates	sugar		
figs			

1. Find the duty on \$3240 worth of porcelain. (Use the fraction equivalent.)
2. Find the duty on \$4800 worth of cinnamon. (Use the decimal equivalent.)
3. Find the duty on \$280 worth of oranges. (Use the fraction equivalent.)
4. Find the duty on \$650 worth of salt. (Use the decimal equivalent.)
5. Find the duty on \$125 worth of ivory. (Use the fraction equivalent.)
6. Find the duty on \$2460 worth of zinc. (Use the decimal equivalent.)
7. Find the duty on \$1500 worth of black pepper. (Use the fraction equivalent.)
8. Find the duty on \$2250 worth of silk. (Use the decimal equivalent.)

## CHALLENGE:

A ship returns to Salem with cargo that has been assessed as follows:

\$3660	iron
\$760	lemons
\$1080	limes
\$4880	coffee
\$2970	tea
\$3450	sugar
\$2000	cinnamon
\$2400	cloves
\$3200	black pepper
\$1555	silk

What is the total amount he may pay in duties?

## Paying Duty Answers

1. \$324
2. \$576
3. \$14
4. \$65
5. \$25
6. \$123
7. \$180
8. \$450

## CHALLENGE:

\$2628



## PAYING DUTY

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$$1/5 \text{ of } \$2500 = \quad \text{("of" with fractions mean "multiply")}$$

Example 2: Find the duty on iron valued at \$600.

Solution: From the chart, the duty on iron is 5%.  $5\% = .05$ .

$$.05 \times \$600 = \quad$$



5%	10%	12%	20%
iron	porcelain	cinnamon	silk
zinc	salt	cloves	ivory
lemons	coffee	nutmeg	jade
limes	tea	ginger	gold dust
oranges	cocoa	black pepper	
dates	sugar		
figs			

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\$2000	cinnamon
\$2400	cloves
\$3200	black pepper
\$1555	silk

What is the total amount he may pay in duties?





## Lesson 3: Trading and Fair Exchange

### Objectives:

- Students will determine a fair exchange for varying amounts of various goods.

### Skills:

- Students will learn to set up proportions (a good problem-solving technique) to find an equivalent ratio given a known rate of exchange.
- Students will be able to cross-multiply to solve the proportions.
- Students will understand, read, use information from a table and construct tables to solve problems.

### Vocabulary:

- Proportion

### Materials:

### Procedure:

1. Distribute worksheet "Trading and Fair Exchange".
2. Introduce proportions: proportions are two equivalent ratios.
3. Use the table on the worksheet to set up proportions.
4. Students cross-multiply to solve the proportions.
5. Challenge Problem: In small teams of three or four students, brainstorm how to solve the problem. Let the quantities in the tables represent units. For example, 120 lbs. of iron equals 1 unit of iron, 24 lbs. of cinnamon equals 1 unit of cinnamon, and so on. 5 units of iron could be traded for 5 units of cinnamon (120 lbs.) or 5 units of cloves (100 lbs.) 4 units of cinnamon (96 lbs.) plus 1 unit of cloves (20 lbs.) or 3 units of cinnamon plus 2 units of cloves (40 lbs.), and so on.

## Trading and Fair Exchange

Let's say anything in Column A of the chart below could be exchanged for anything in Column B. For example, 24 lbs. of cinnamon could be traded for 120 lbs. of iron or 50 barrels of flour or 150 bales of cotton, etc. The 120 lbs. of iron, 50 barrels of flour or 150 bales of cotton could also be traded for 20 lbs. of cloves or 36 lbs. of coffee, and so on. Use the information from the table and set up proportions to solve the problems below. (Examples are hypothetical.)

A	B
24 lbs. of cinnamon	120 lbs. iron
20 lbs. of cloves	200 pcs. of wooden ware
45 lbs. of black pepper	50 barrels of flour
36 lbs. of coffee	150 bales of cotton
32 lbs. of sugar	54 sacks of corn
12 lbs. of tea	48 sacks of rice
100 lbs. of salt	72 bundles of tobacco
18 bolts of silk	60 barrels of dried fish

**Example:** How many lbs. of cloves could you trade for 60 bales of cotton?

**Solution:** Set up a proportion:  $\frac{150 \text{ bales cotton}}{60 \text{ bales cotton}} = \frac{20 \text{ lbs. cloves}}{Y}$

**Cross-product:**  $60 \times 20 = 150Y$

$$1200 = 150Y$$

$$Y = 8, \text{ answer}$$

8 lbs. of cloves can be traded for 60 bales of cotton

Here are some for you try:

1. How many bolts of silk could you trade for 500 bundles of tobacco?
2. How many sacks of corn would you need to trade for 105 lbs. of black pepper?
3. How many lbs. of coffee could you trade for 225 barrels of dried fish?
4. How many pounds of iron would you need to trade for 72 lbs. of sugar?

5. How many pounds of cinnamon could you trade for 84 sacks of rice?
6. How many barrels of flour would you need to trade for 42 lbs. of tea?

**CHALLENGE:** Work in groups of three or four to solve the next two problems. Each problem has several answers. (Hint: making a table would be a could problem-solving strategy here.)

- A. How much salt and pepper could be traded for 600 lbs. of iron?
- B. How much much corn and rice would be a fair exchange for 72 lbs. of cinnamon and 80 lbs. of cloves?

### Trading and Fair Exchange Answers

1. 125 bolts of silk
2. 126 sacks of corn
3. 135 lbs. of coffee
4. 270 lbs. of iron
5. 42 lbs. of cinnamon
6. 175 barrels of flour

### CHALLENGE:

A. Solution: Think of 120 lbs. of iron as 1 unit of iron (I), 100 lbs. of salt as 1 unit of salt (S), and 45 lbs. of pepper as 1 unit of pepper (P). The units are all equivalent since

120 lbs. of iron is worth 100 lbs. of salt or 45 lbs. of pepper. 600 divided by 120 is 5,

so we need 5 units of salt and pepper to equal 5 units of iron. We could have 1 unit of

salt plus 4 units of pepper, 2 units of salt and 3 units of pepper, 3 of salt and 2 of

pepper, or 4 of salt and 1 of pepper.

salt	iron	pepper
1@ 100 lbs=100 lbs. S=120 lbs. I	$120 + 480 = 600$ lbs.	4@ 45 lbs=180 lbs. P=480 lbs. I
2@ 100 lbs=200 lbs. S=240 lbs. I	$240 + 360 = 600$ lbs.	3@ 45 lbs=135 lbs. P=360 lbs. I
3@ 100 lbs=300 lbs. S=360 lbs. I	$360 + 240 = 600$ lbs.	2@ 45 lbs= 90 lbs. P=240 lbs. I
4@ 100 lbs=400 lbs. S=480 lbs. I	$480 + 120 = 600$ lbs.	1@ 45 lbs= 45 lbs. P=120 lbs. I

B. This is similar to the problem above, but a little more complex. 72 lbs. of cinnamon would be equal to 3 units, and 80 lbs. of cloves would be equal to 4 units. So we need combinations of corn and rice that equal 7 units. The units combinations are listed below. It is up to the reader to compute the various amounts (e.g., 2 corn units = 108 sacks corn + 5 rice units = 240 sacks rice).

corn units	rice units	cinnamon units	clove units
1	6	3	4
2	5	3	4
3	4	3	4
4	3	3	4
5	2	3	4
6	1	3	4



# TRADING AND FAIR EXCHANGE

Let's say anything in Column A of the chart below could be exchanged for anything in Column B. For example, 24 lbs. of cinnamon could be traded for 120 lbs. of iron or 50 barrels of flour or 150 bales of cotton, etc. The 120 lbs. of iron, 50 barrels of flour or 150 bales of cotton could also be traded for 20 lbs. of cloves or 36 lbs. of coffee, and so on. Use the information from the table and set up proportions to solve the problems below. (Examples are hypothetical.)

A	B
24 lbs. of cinnamon	120 lbs. iron
20 lbs. of cloves	200 pcs. of wooden ware
45 lbs. of black pepper	50 barrels of flour
36 lbs. of coffee	150 bales of cotton
32 lbs. of sugar	54 sacks of corn
12 lbs. of tea	48 sacks of rice
100 lbs. of salt	72 bundles of tobacco
18 bolts of silk	60 barrels of dried fish

**Example:** How many lbs. of cloves could you trade for 60 bales of cotton?

**Solution:** Set up a proportion:  $\frac{150 \text{ bales cotton}}{60 \text{ bales cotton}} = \frac{20 \text{ lbs. cloves}}{Y}$

**Cross-product:**  $60 \times 20 = 150Y$   
 $1200 = 150Y$   
 $Y =$   
 lbs. of cloves can be traded for 60 bales of cotton

Here are some for you try:

1. How many bolts of silk could you trade for 500 bundles of tobacco?
2. How many sacks of corn would you need to trade for 105 lbs. of black pepper?
3. How many lbs. of coffee could you trade for 225 barrels of dried fish?



4. How many pounds of iron would you need to trade for 72 lbs. of sugar?
5. How many pounds of cinnamon could you trade for 84 sacks of rice?
6. How many barrels of flour would you need to trade for 42 lbs. of tea?

**CHALLENGE:** Work in groups of three or four to solve the next two problems. Each problem has several answers. (Hint: making a table would be a good problem-solving strategy here.)

- A. How much salt and pepper could be traded for 600 lbs. of iron?
- B. How much much corn and rice would be a fair exchange for 72 lbs. of cinnamon and 80 lbs. of cloves?

## Open Response Questions

1. If Nathaniel Bowditch had been born into vastly different economic, historical, or geographical circumstances, how would his life or contributions have changed?
2. How would Nathaniel's life have been different if he had been able to attend Harvard?
3. Describe how you would feel if you were forced to live away from home at the age of 12, indentured for the next 9 years, and never given an opportunity to go to school with your friends?
4. What do you think are the qualities of a genius? Based on your answer, was Nathaniel Bowditch a genius? Why or why not?
5. If Bowditch had been able to converse with Sir Isaac Newton, how would that discussion have influenced the direction of Nathaniel's life or his contribution to mathematics and to science?
6. Like Bowditch, how could you explain a problem to someone younger than you, for instance, in elementary school? Select a problem and write down the steps you would take to teach that student. Then, mentor a younger student, correcting your teaching method as you go along so that it works better the next time.
7. Compare your town's population and pattern of growth with that of Salem's during the last two hundred years. What could have caused these patterns? Can you predict future trends? Include immigration patterns and economic factors if possible.
8. Explain in your own words the following quote about Bowditch,  
*His intuitive mind sought and amassed knowledge, to impart it to the world in more easy forms.*



# Glossary

**accuracy**

exactness; precision

**actuary**

a statistician who uses probabilities to compute insurance risks and premiums

**approximate**

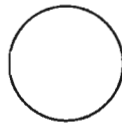
very close to

**bookkeeper**

one who records the accounts and transactions of a business

**circle**

the set of all points in a plane that are a fixed distance from a fixed point (center)

**congruent**

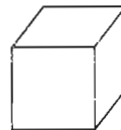
same size and same shape

**consecutive**

in a row, one right after the other

**cube**

a solid (three-dimensional) figure with six congruent, square faces; a number raised to the third power

**ellipse**

oval

**exponent**

the power to which a (base) number is raised. In the example below, 3 is the exponent and 5 is the base. 5 is used as a factor 3 times.

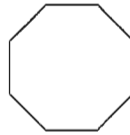
$$5^3 = 5 \times 5 \times 5 = 125$$

**hexagon**

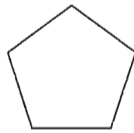
a six-sided closed plane figure

**octagon**

an eight-sided closed plane figure

**pentagon**

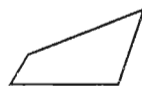
a five-sided closed figure

**plane**

a two-dimensional flat surface---i.e., a surface with length and width, but no thickness

**quadrilateral**

any four-sided closed plane figure

**quarter**

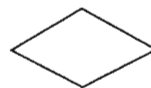
circle-1/4 of a circle

**rectangle**

a closed plane figure with four equal angles ( $90^\circ$ ). Opposite sides are equal and parallel.

**rhombus**

a closed plane figure with four equal sides, but not necessarily four equal angles. A rhombus has a diamond shape.



**semicircle**

1/2 of a circle

**square**

a closed plane figure with four sides of equal length and four angles of equal measure ( $90^\circ$ )

**square number**

a number which is the result of multiplying another number by itself, and which can be represented by a square arrangement of dots. For example, 25 is a square number because it is the result of multiplying 5 by itself (i.e.,  $5 \times 5 = 25$ ) and can be represented by a square arrangement of dots.

**supercargo**

an officer on a merchant ship who has charge of the goods traded

**surveyor**

one who measures angles and distance to determine boundaries, area, elevation.

**symmetry**

balance; exact correspondence of design and form on opposite sides of a dividing line or around a central point

**transaction**

any kind of business activity, such as a sale or a purchase, a debit or a credit

**trapezoid**

a closed plane figure with four sides, two of which are parallel and two of which are not

**triangle**

a three-sided closed plane figure



## Terms of the Sea

1. **Ash Breeze** – Becalmed. Under this condition, a ship's boats were put out to tow her by rowing. Oars were made of ash wood.
2. **Astrolabe** – A predecessor to the sextant; an instrument for measuring the altitude of celestial bodies. From the Greek word *astrolabeon*, meaning star taking.
3. **Bowditch** – A household word among navigators; a navigation textbook and navigation tables still in use by the U.S. Navy.
4. **Chronometer** – A highly accurate clock used in navigation. Used to determine accurate longitude readings. A British carpenter named John Harrison designed and built the first truly accurate chronometer in the mid-18th century.
5. **East India Company** – An organization set up by Queen Elizabeth in 1600 to trade in India. Given monopolistic power, the company controlled India, acting as the governmental authority for British possessions in the Far East. The "John Company" as it was known, remained in power until 1857 after which it was dissolved by the British government following the disastrous Sepoy Rebellion.
6. **East Indiaman** – A large, heavily armed merchant ship built for the East Indies trade.
7. **Indentured Servant** – A contract for labor for a specified time period.
8. **Letter of Marqué** – a royal license, authorizing a vessel to privateer under a recognized flag.
9. **Lunar** – Term for taking a celestial measurement of the moon when determining longitude.
10. **Magazine** – A special hold on naval and merchant vessels in which ammunition is stowed.
11. **Pitch** – A mixture of tar and other substances used to caulk and preserve the wood and cordage of sailing vessels.
12. **Privateer** – A privately owned vessel of war, furnished with a commission – Letters of Marqué.
13. **Ropewalk** – A place of rope manufacture. Typically a long, narrow building in which rope is twisted to form cordage for sailing ships.

14. **Sail loft** – A place of sail manufacture, typically located on the top floor of warehouses in any port.
15. **Ship chandlery** – The shoreside supermarket for all nautical equipment.
16. **Slaver** – A vessel that transports slaves.
17. **Supercargo** – An officer or a supernumerary on a merchant ship who was in charge of trading – a sea-going merchant.
18. **Typhoon** – A strong tropical storm in the Pacific.





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## **NATHANIEL BOWDITCH 1773-1838**

Abridged Version, To obtain an unabridged version with references contact Susan Bowditch at The House of the Seven Gables.

### **ORIGINS OF THE BOWDITCH FAMILY IN ENGLAND, 1170-1670**

The family name is said to have originated in Chardstock, County Dorset, England in the early 12th century. Bowditch is thought to mean boundary-ditch because the family's holdings and houses began around an ancient pre-historic curved ditch. Ditches were dug by tribes to protect themselves from their neighbors. The curve of the ditch may have inspired the "Bow" in the name.

### **NATHANIEL BOWDITCH, THE EARLY YEARS, 1773-1784**

Nathaniel was born in Salem, the 4th child of Habakkuk and Mary Ingersoll Bowditch. The family moved to Danvers when Nathaniel was two. He and his siblings attended the Dame's School located directly across from his home. Tuition is assumed, and since the family was of humble means it is not known how they afforded the schooling for the children, nor is it known how long or how often the children actually attended school.

At the age of seven, Nathaniel and his family moved back to Salem where life became very difficult financially. His mother was to have favored Nathaniel and was highly influential in molding his character and his high regard for truth above all values.

At the age of seven or eight, Nathaniel and his older brother Hab attended Master Watson's school. He looked forward to school and begged to be given challenging mathematical problems reserved for older students. The master resisted Nathaniel's efforts, but having got Nathaniel's father's permission, decided to give him a problem he was sure he could not solve. When Nathaniel came back the next day with the correct answer, Master Watson accused him of getting someone else to solve the problem. Had not Nathaniel's older brother Hab intervened, Master Watson would have whipped Nathaniel for lying. The bitterness accompanying this incident was felt by Nathaniel for the rest of his life, as much an accusation against his integrity as his mathematical skills.

Due to hard times, in 1783 Nathaniel's father withdrew him from school so that he could help in his cooper's shop. Nathaniel hated leaving school and barrel making did not agree with him due to his health and stature. After two years, his father enrolled him in a bookkeeping course and then inden-

tured Nathaniel to the ship chandlery shop of Ropes and Hodges. During this period Nathaniel's mother died. When Ropes and Hodges went out of business in 1790, Nathaniel continued his apprenticeship with Samuel Curwen Ward, also a ship chandler and grocer.

Nathaniel discovered algebra through an older brother at the age of fourteen. He was highly motivated and excited at the notion of calculating with letters as well as numbers. It was through this method that he was to learn navigation. The basics of navigation were thought to have been taught to him by an English sailor and a local mariner, George Chapman. When the sailor left him to return to England he said, "Nat my boy, go on studying as you do now, and you will be a great man one of these days." Nathaniel never forgot those encouraging words and continued to be self directed, motivated, and a self-taught individual, thus achieving more than the sailor ever could have imagined.

At the age of 15 or 16, Nathaniel constructed an odd barometer and compiled a navigational almanac that identified reference points at sea by fixing a ship's position north or south of the equator. At 17 he decided to learn Latin so that he could read Newton's *Principia*, which dealt with the workings of the universe. By the age of 21 he completed the translation and detected an error in the calculations. This error was noted in correspondence that Nathaniel had with a mathematics professor at Harvard. The letter was discounted as being from a young upstart. Nathaniel later published this error.

By the time he was 19 Nathaniel knew enough about navigation to design and create a quadrant, the most common instrument of navigation for the time. (It is presently in the custody of the Peabody Essex Museum.) At the age of 19 he also constructed a wooden sundial.

Nathaniel created his own college experience by reading what is known today as Salem's Philosophical Library. This scientific library, captured aboard the English ship *Mary* and belonging to Dr. Richard Kirwan, a noted Irish scientist, was acquired by privateers out of Beverly and sold to intellectuals in Salem. These readings provided Nathaniel with an education that far exceeded Harvard College of the day. Nathaniel was also self taught with regard to surveying skills and foreign languages.

## NATHANIEL BOWDITCH AS A YOUNG MAN, 1795-1803

For most of the next nine years Bowditch was at sea, engaged in five different voyages.

The voyages:

- I.     *The Henry*, January 11, 1795-January 11, 1796, to the Isle of Bourbon and Isle of France
- II.    *The Astrea*, March 15, 1796-May 22, 1797, to Portugal and the Philippines
- III.   *The Astrea*, August 21, 1798-April 6, 1799, to Spain
- IV.    *The Astrea*, July 23, 1799-September 15, 1800, to Indonesia and the Philippines
- V.     *The Putnam*, November 21, 1802-December 25, 1803, to Sumatra

During these voyages Bowditch made notes on slavery and tried his ideas on celestial navigation, thus training crew members to do "lunars." Lunars involved taking three simultaneous sights on the moon and a fixed star, or the sun, and calculating the angular distance between them; then, with reference to a nautical almanac, obtain Greenwich time and hence the exact longitude. Bowditch also spent a great deal of time correcting the book by John Hamilton Moore, *The Practical Navigator*. His extensive journal-writing on these voyages reflected on the cultures of the people and the ports he visited.

Bowditch married Elizabeth Boardman in March 1798, but this union was ill fated as Elizabeth died in October while Bowditch was in Spain.

In May 1799, Bowditch was elected to the American Academy of Arts and Sciences, an important and impressive honor for a self-educated man.

In 1800 Bowditch married Mary Ingersoll, his first cousin. They continued to live with Mrs. Boardman, his first wife's mother, as Elizabeth his first wife and Mary, had been close.

The East India Marine Society was founded in Salem in 1799. All members had to be captains or supercargoes (businessmen) who had traversed the Cape of Good Hope or Cape Horn. Bowditch was asked to be a member. Through this union he was instrumental in insisting that each crew member be provided with a journal. Following each voyage the completed journals were to be available to the Society to aid future voyages. Another requirement of the Society was to bring back art forms plus natural and cultural artifacts representative of the places to which the ships had sailed. (This



enormous collection housed above the Asiatic Bank in Salem eventually became the Peabody and then the Peabody Essex Museum.)

While at sea, Bowditch published *The New American Practical Navigator* in 1802 in both Europe and America. It became the most important book on navigation ever published. It is still published and used today, in revised form, by The United States Navy.

Also in 1802, while windbound in Boston, Bowditch decided to visit Cambridge. By coincidence on this day, commencement exercises were being held at Harvard. As the honorary degrees were being conferred (in Latin) Bowditch thought he heard his name and soon discovered that he had been awarded an Honorary Master of Arts degree. This honor remained till his dying day the most touching and significant of all awards received during his lifetime.

Bowditch finally set sail on *The Putnam* as master and part owner. During this voyage he began translating the latest book on astronomy, *Mécanique Celeste* by Pierre La Place. Though written in French, the translation was a minor challenge, as Bowditch was self taught in Latin, French, Spanish, Portuguese, and German.

Bowditch returned to Salem aboard *The Putnam* on Christmas day under severe fog conditions. Using his navigational genius he was able to steer the Putnam into Salem Harbor as if it were a clear summer day. At 4:00 on December 25th he had sighted Cape Ann and with that sighting, plus the sighting of the sun two days earlier and one momentary glimpse of the light on Bakers Island, Bowditch was able to steer his way through the fog.

### **NATHANIEL BOWDITCH, THE MIDDLE YEARS, 1804-1823**

Bowditch became the president and the first actuary of Essex Fire and Marine Insurance Company. With the risks attendant to ships, crews, and cargoes, insurance companies were very much needed. Such work appealed to Bowditch, as it afforded him an opportunity to continue calculating. By day he was an insurance calculator and by night an astronomer.

During the years from 1804-1806 he was asked to survey the harbors of Salem, Marblehead, Beverly, and Manchester and compute the height of Mt. Washington.

In 1805 the Bowditches had their first child and moved from the Boardman house on Washington Square to Chestnut Street where they shared a house with the Hodges, occupying the east side of the house. The Bowditches had four children before they were able to buy their own home in 1811.

In 1806 he observed an eclipse of the sun, which he later wrote about in the *Memoirs of the American Academy of Arts and Sciences*. During the same year he wrote about the variation of the magnetic needle of the compass. He later wrote about the orbits of comets that appeared in 1807, 1811, and 1819. Bowditch has been credited with publishing a total of 31 scientific articles.

Though an excellent teacher at sea, Bowditch did not like speaking in public and declined offers to teach at Harvard, the University of Virginia, and West Point.

In 1810 Bowditch was made Overseer of Harvard College and in 1816 he was awarded an Honorary Doctor of Laws at Harvard. From 1814 to 1817, he completed *The Translation and Commentary of Mécanique Celeste* but did not publish it until he could afford to do so personally between 1829 and 1838. His thorough research and interest doubled the pages of the original document. By 1820 Bowditch had become president of the East India Marine Society.

In 1823 the Bowditches left Salem to live in Boston.

## **NATHANIEL BOWDITCH, THE LATER YEARS, 1823-1838**

At age fifty, Bowditch took responsibility as Actuary of the Massachusetts Hospital and Life Insurance Company, plus President of the Commercial Insurance Company (Fire and Marine), both in Boston.

Bowditch made the insurance company prosper by calculating new insurance tables, simplifying bookkeeping and weathering panics and inflation.

The epitome of Bowditch's career as a scholar of both mathematics and astronomy lies not only in *The New American Practical Navigator* but also in *The Translation and Commentary of Mécanique Celeste*. Bowditch thus earned entry into almost every scientific society in the world. It is considered by many to be the best follow-up to Newton's *Principia*. Bowditch's genius with respect to his translation of La Place's book does not lie in his own discoveries, but in his ability to analyze, correct and make readable difficult scientific content. This in the end was his most important contribution to the world. The amazing thing is that Bowditch accomplished these translations while working a full time job, caring for a large family and without the patronage that was prevalent in Europe at this time.

On March 16, 1838, Bowditch died at the age of 65 with his family around him. His library, considered the best mathematical library in the United States, was eventually given to The Boston Public Library.

